Estimating the Welfare Cost of Labor Supply Frictions

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1 Introduction

Individuals are often unable to work as much as they would prefer in an ideal world due to various factors such as (1) high costs of changing jobs (Varejão and Portugal, 2007), (2) employer mandates on how many hours per week they must work (Dickens and Lundberg, 1993b), (3) a lack of desirable jobs, and/or (4) misperceptions about the value of their time (DellaVigna, 2009). This paper seeks to explore the magnitudes and quantify the welfare cost of these sorts of labor market frictions.

More specifically, this paper seeks to quantify how much people would be willing to pay to (hypothetically) remove frictions which impede them from supplying their optimal amount of labor. For instance, how costly is it (from the individual's perspective) for a person who would like to work 35 hours/week but can only find a job that requires 40 hours/week? We begin by analyzing a standard, static labor supply model in which individuals have utility over consumption and hours worked. Individuals' maximize utility with some choice of ideal (i.e., frictionless) hours h^*, but frictions lead individuals to end up working a different amount of actual hours with frictions, h^F. We assume to begin with that agents correctly perceive wages, taxes, and their utility function, but otherwise do not take a stance on the type of frictions that generate this deviation: some examples which are consistent with our framework are fixed adjustment costs, search costs, or the presence of limited choice sets. We refer to these sorts of frictions collectively as "adjustment frictions". First, we show how to recover an estimate of the willingness-to-pay (WTP) to remove adjustment frictions using two empirical objects: (1) the percentage difference between ideal and actual hours and (2) the Hicksian elasticity of ideal hours with respect to the after-tax wage rate. The core intuition, which is similar to insights in Akerlof and Yellen (1985), Mankiw (1985), and Chetty (2012) is that the Hicksian elasticity encodes the curvature of utility. This estimate is valid up to third order terms in the percentage difference between ideal and actual hours. For very large frictions (i.e., when the percentage difference between ideal and actual hours is very big), we derive revealed preference arguments to bound the size of the WTP to remove frictions, thereby mitigating the impact of third and higher order terms on our estimates. Importantly, our framework is quite general insofar as we make no structural assumptions on the utility function or the type of frictions agents face.

Next, we empirically implement this methodology to recover the cost of adjustment frictions. Towards this end, we use data from the National Survey of the Changing Workforce (NSCW) in the U.S. and the German Socio-Economic Panel (GSOEP) which contain data on how many hours individuals work as well as how many hours they would ideally like to work. We document a considerable discrepancy between these two variables: on average across the population, ideal hours of work is XX% (YY%) different from their actual hours of work in the NSCW (GSOEP). Next, we leverage the panel nature of the GSOEP to estimate the Hicksian elasticity of ideal hours with respect to the after-tax wage rate using tax variation in the 1990s using a difference-in-difference strategy similar to Gruber and Saez (2002) and Kopczuk (2005). While our estimates of this elasticity are noisy, the point estimates are consistent across specifications (≈ 0.13) and, most importantly, every specification can reject the hypothesis that the ideal hours elasticity is larger than one.[1] We then put these pieces together to compute the cost of adjustment frictions. We find that average WTP to remove hours frictions is around 20% of income in the NSCW and GSOEP if the elasticity of ideal hours is 0.13; if this elasticity is 1 then the average WTP to remove hours frictions is 7% (4%) in the NSCW (GSOEP). Hence, the first key empirical takeaway is that the cost of adjustment frictions is relatively large for any reasonable value of the ideal hours elasticity.

The next section of the paper shows how to recover the cost of adjustment frictions in more general settings. In particular, we show how our analysis can be extended to allow for an additional unobservable labor supply decision (effort per hour), many hours worked decisions (e.g., multiple earners per household), and dynamic decision making environments with savings as well as endogenous and stochastic wage growth. There are two high-level takeaways: (1) we can still recover the cost of adjustment frictions using ideal hours, actual hours, and elasticities (although some of these more complex models require additional elasticities beyond just the Hicksian elasticity of ideal hours) and (2) empirically, the cost of adjustment frictions is still relatively large even accounting for these additional complexities.

Next, we consider the cost of misperception frictions, focusing specifically on misperceptions of the tax schedule. We first show how to recover the cumulative cost of adjustment frictions and misperceptions using data on ideal hours, actual hours, the size of tax misperceptions, and

[1] To the best of our knowledge, no paper has attempted to estimate the ideal hours elasticity (i.e., the hours elasticity in absence of frictions). However, several papers (e.g., Kleven and Waseem (2013), Gelber et al. (2020), Tazhitdinova (2020)) have estimated the frictionless earnings elasticity, which we typically expect to be larger than the frictionless hours elasticity. These papers also find that the frictionless earnings elasticity is less than 1.

the Hicksian elasticity of ideal hours. We then use data on ideal hours and actual hours from the NSCW along with data on the size of tax misperceptions from Rees-Jones and Taubinsky (2020) to estimate the cumulative cost of adjustment frictions and tax misperceptions. There are two findings. First, adjustment frictions contribute far more to the cumulative cost of frictions than do misperceptions of the tax schedule for any reasonable value of the Hicksian ideal hours elasticity. Second, while the cost of adjustment frictions decreases with the Hicksian ideal hours elasticity (because higher elasticities imply less curvature in utility), the cost of tax misperceptions actually increases with the Hicksian ideal hours elasticity (because people change their behavioral more in response to misperceptions). As a result, we find that the cumulative cost of adjustment frictions and misperceptions is at least 10% of income *regardless of the value of the Hicksian ideal hours elasticity.*

We are certainly not the first paper to explore the impacts of labor market frictions. A number of papers have proposed that adjustment costs and hours constraints have important impacts of labor market outcomes (e.g. Cogan (1981), Ham (1982), Altonji and Paxson (1988), Dickens and Lundberg (1993a)). More recently, a number of papers have documented the presence of labor market frictions and illustrated the consequent fiscal impacts (Chetty et al. (2011), Chetty et al. (2013b), Chetty et al. (2013a), and Kleven and Waseem (2013)); however fewer papers have attempted to explore the welfare costs of these frictions. Much of the recent investigation into the cost of frictions comes from (a lack of) bunching at kink points in tax schedules. For example, Søgaard (2019) uses bunching evidence to estimate the cost of adjustment frictions for Danish students, finding a cost of optimization frictions around 2-3% of income. Gelber et al. (2020) estimate the size of adjustment cost frictions using changes in bunching in the context of the Social Security Earnings Test, finding that adjustment costs are around 0.5-2% of mean earnings. Gudgeon and Trenkle (2024) apply this same method to a tax notch in Germany, finding an average cost of adjustment cost of €400-500, which is around 1% of household income. In terms of the costs of misperceptions, Rees-Jones and Taubinsky (2020) find that the welfare cost of tax misperceptions is around 0.5% of income.

Relative to previous work, our first contribution is methodological: we develop a framework to estimate the cost of frictions that places few assumptions on the functional form of utility and can accommodate many different sorts of frictions (discrete choice sets, adjustment costs, demand side constraints, search costs, and misperceptions). Moreover, we show that our approach can be extended to incorporate many realisms (heterogeneous effort per hour, joint labor supply decisions, and dynamic decision making environments). Previous work has typically focused on estimating the cost of a specific type of friction (e.g., adjustment costs or limited choice sets or misperceptions) and typically makes stronger functional form assumptions (i.e., quasi-linear iso-elastic utility).

Our theoretical results illustrate the importance of attempting to account for many sources of frictions simultaneously: the cost of frictions grows *quadratically* with the deviation from optimal hours induced by frictions. Hence, three different frictions (e.g., adjustment costs, discrete choice sets, and misperceptions) which each lead to successive 1% deviations from optimal hours generate a cumulative cost of frictions that is 9 times as large as each of these frictions individually. While previous papers (e.g., Gelber et al. (2020), Gudgeon and Trenkle (2024), Chetty (2012), Rees-Jones and Taubinsky (2020)) all illustrate particular types of frictions that generate a cost valued at $\approx$ 1% of income, we find that the cumulative cost of adjustment frictions and tax misperceptions is closer to 10% of income.

From a policy perspective, our core finding that labor market frictions are substantial suggest that there may be considerable value in policy reforms that reduce these frictions. For example, understanding the cost of frictions is crucial for evaluating (1) laws and regulations that increase the availability of gig work, (2) regulations that impose fixed costs on firms (e.g., benefit mandates for individuals who work a certain amount or compliance costs that scale with the number of workers but not amount worked by each worker), (3) proposals for government employment guarantees, (4) laws on paid-time-off, or (5) initiatives that improve tax salience.

Secondarily, this paper also contributes to the literature on estimating structural parameters in the presence of frictions (e.g., Chetty (2012), Kleven and Waseem (2013), Gelber et al. (2020), Gudgeon and Trenkle (2024), Tazhitdinova (2020)). In addition to estimating a frictionless hours elasticity in the context of German tax reforms in the 1990's (which is noisy but almost certainly less than 1), we also illustrate how our work can be combined with the insights of Chetty (2012) to set-identify structural (frictionless) elasticities from reduced from estimates. Precisely, our methodology provides a way to estimate the cost of frictions, which is an input into the bounding exercise of Chetty (2012) (which has previously just been assumed rather than estimated). Combining these two methodologies, we show that because our estimated cost of frictions is so substantial, that almost any structural elasticity can be consistent with any observed elasticity. In other words, because we estimate such a large cost of frictions, we can not learn much about the structural labor supply elasticity from the observed elasticity using the methodology of Chetty (2012).[2]

Finally, in the interest of transparency, our methodology has three key downsides relative to prior work: (1) we estimate a second order approximation for the cost of frictions which can be inaccurate for individuals with very large frictions (we mitigate this issue by capping

[2] Note, this strengthens the overall message of Chetty (2012) that one can reconcile seemingly disparate elasticity estimates in the presence of frictions.

the cost of frictions using revealed preference bounds, but it is nonetheless a limitation), (2) our analysis only estimates frictions for employed individuals (we would need counterfactual wages to apply our results for unemployed individuals) and (3) our approach inherently relies on (unincentivized) survey data because we need estimates of ideal hours; this is an important limitation compared to other approaches (e.g., Gelber et al. (2020)) which can be applied with administrative data.

2 Adjustment Frictions Theory

Our goal is to understand how much worse off individuals are as a result of their actual labor supply deviating from their ideal labor supply due to frictions. We will separately analyze the impacts of two different kinds of frictions: adjustment frictions and misperceptions of the tax schedule. Adjustment frictions should be thought of as constraints which prohibit individuals from supplying their ideal amount of labor but are *not* related to misperceptions. For instance, limited choice sets arising from demand side constraints, search costs, and fixed costs of changing jobs would all fall under our broad definition of adjustment frictions.

2.1 Adjustment Frictions in a Baseline Labor Supply Model

We begin with a standard labor supply model in which individuals have preferences over consumption, c, and hours worked, h. Individuals have a smooth utility function:

$$u(c, h)$$

Agents face a (potentially) non-linear tax schedule $T(wh)$ where w is their hourly wage:

$$c = wh - T(wh) \tag{1}$$

To begin with, we assume that agents correctly perceive the tax schedule; we discuss how the results change with misperceptions in Section 5. Let h^* denote the solution which maximizes $u(c, h)$ subject to the budget constraint, Equation (1); we will refer to h^* as "ideal hours". However, suppose that agents face some sorts of adjustment frictions. For instance, perhaps agents maximize $u(c, h)$ s.t. Equation (1) and the additional constraint that $h \in H^F$ where H^F denotes some limited choice set. Or agents maximize $u_F(c, h) = u(c, h) - \kappa \mathbb{1}[h \neq \tilde{h}]$ s.t. Equation (1) where κ denotes a fixed cost that agents must pay to deviate from a status quo hours $\tilde{h}$. As a result of adjustment frictions, the agent ends up supplying h^F hours and thus consumes $c(h^F) = wh^F - T(wh^F)$; we will refer to h^F as "observed hours". We are interested in understanding the cost of these adjustment frictions. Precisely, we will be interested in estimating the parameter $\delta(h^F)$ which represents the willingness-to-pay (WTP) to remove frictions defined implicitly by the following expression:

$$u(c(h^F) + \delta(h^F), h^F) = u(c(h^*), h^*) \tag{2}$$

$\delta(h^F)$ is the amount of money required to compensate an individual for frictions which prevent them from choosing their optimal hours, h^*. Henceforth, we will refer to $\delta(h^F)$ as the "cost of frictions" and/or the "WTP to remove frictions", recognizing that $\delta(h^F)$ is a money-metric. First, we will show how to (approximately) measure $\delta(h^F)$ in terms of two objects: the percentage difference between h^F and h^* as well as the compensated elasticity of ideal (i.e., frictionless) hours. Towards this end, let us explore how $\delta(h)$ changes with h as hours worked moves away from h^* by applying the implicit function theorem to Equation (2) to calculate:

$$\left.\frac{d\delta}{dh}\right|_{h^*} = -\frac{u_c(c(h^*), h^*)w(1 - T'(wh^*)) + u_h(c(h^*), h^*)}{u_c(c(h^*), h^*)} = 0 \tag{3}$$

where the second equality follows from the FOC which is satisfied at h^* assuming $T(wh)$ is differentiable at h^*. Equation (3) is a consequence of the envelope theorem and tells us that it is not costly for agents to experience frictions that move them infinitesimally far away from their optimal choice of hours. As a result of Equation (3), we can use the fundamental theorem of calculus twice to deduce:

$$\delta(h) = \int_{h^*}^{h} \frac{d\delta}{dh}(s)ds = \int_{h^*}^{h}\int_{h^*}^{s} \frac{d^2\delta}{dh^2}(t)dtds \tag{4}$$

Hence, in order to understand the cost of frictions, we require an estimate of the second derivative of frictions as we move away from h^*. Applying the implicit function theorem to Equation (3), we can get an expression for $\frac{d^2\delta}{dh^2}|_{h^*}$ (see Appendix A.1 for the proof):

$$\left.\frac{d^2\delta}{dh^2}\right|_{h^*} = -\frac{u_{cc}w^2(1 - T')^2 - u_c w^2 T'' + 2u_{ch}w(1 - T') + u_{hh}}{u_c} \tag{5}$$

Note that the (strict) second order condition for optimality of h^* requires that $u_{cc}w^2(1-T')^2 - u_c w^2 T'' + 2u_{ch}w(1-T') + u_{hh} < 0$ so that $\frac{d^2\delta}{dh^2}|_{h^*} > 0$. Equation 5 tells us that $\frac{d^2\delta}{dh^2}|_{h^*}$ is equal to the curvature in $u(wh - T(wh), h)$ w.r.t. h divided by the marginal utility of consumption. We will now show that $\frac{d^2\delta}{dh^2}|_{h^*}$ can be expressed in terms of the Hicksian elasticity of ideal hours of work w.r.t. the tax rate, denoted ξ_{h^*}. When faced with a linear tax schedule ξ_{h^*} is the elasticity of h^* w.r.t. the keep rate holding utility constant: $\xi_{h^*} \equiv \frac{\partial h^*}{\partial(1-T')}\frac{1-T'}{h^*}|_u$ (we define this elasticity more generally when agents face a non-linear tax schedule in the proof of Lemma 1 in Appendix A.2). We have:

Lemma 1. *$\frac{d^2\delta}{dh^2}|_{h^*}$ is related to the Hicksian ideal hours elasticity ξ_{h^*} via:*

$$\left.\frac{d^2\delta}{dh^2}\right|_{h^*} = \frac{w(1 - T')}{\xi_{h^*} h^*} \tag{6}$$

Lemma 1 tells us that $\frac{d^2\delta}{dh^2}|_{h^*}$, which equals the curvature in $u(wh - T(wh), h)$ w.r.t. h

divided by marginal utility of consumption, is inversely proportional to the Hicksian labor supply elasticity. The intuition is that utility functions with higher curvature must generate smaller elasticities because people have a stronger preference to choose a given hours worked and are therefore less willing to change their hours substantially with the tax rate. This brings us to our first result:

Proposition 1. *Suppose that* $\frac{d^3\delta}{dh^3} \approx 0$*. The WTP to remove frictions expressed as a fraction of optimal income* wh^* *can be approximated as:*

$$\frac{\delta(h^F)}{wh^*} \approx (1 - T'(wh^*)) \frac{\left(\frac{h^F - h^*}{h^*}\right)^2}{2\xi_{h^*}} \tag{7}$$

Proof. Starting from Equation (4), using the assumption that $\frac{d^3\delta}{dh^3} \approx 0$ so that $\frac{d^2\delta}{dh^2}$ is roughly constant at $\frac{w(1-T')}{\xi_{h^*} h^*}$ by Lemma 1 between h^* and h yields:

$$\delta(h^F) = \int_{h^*}^{h^F} \int_{h^*}^{s} \frac{d^2\delta}{dh^2}(t) dt ds \approx \int_{h^*}^{h^F} \int_{h^*}^{s} \frac{w(1-T')}{\xi_{h^*} h^*} dt ds$$
$$= \int_{h^*}^{h^F} \frac{w(1-T')}{\xi_{h^*} h^*}(s - h^*) ds = \frac{w(1-T')}{2\xi_{h^*} h^*}(h^F - h^*)^2$$

□

Hence, one can recover an estimate of the cost of frictions with an estimate of the Hicksian elasticity of ideal hours with respect to the tax rate, ξ_{h^*}, along with the extent to which observed hours deviate from ideal hours in percentage terms: $\frac{h^F - h^*}{h^*}$. Proposition 1 captures the idea that costs of deviation from an optimum are a function of the second derivatives of utility as discussed in Akerlof and Yellen (1985) or Mankiw (1985). Proposition 1 is conceptually the same as Lemma 1 from Chetty (2012) with the additions that Proposition 1 allows for non-linear tax schedules and non-quasilinear utility. Proposition 1 should thus be viewed as a baseline for recovering the cost of frictions; the primary theoretical contribution of the present paper is to illustrate how the logic of Proposition 1 can be extended to much more general settings (Sections 4 and 5) as well as how to mitigate errors arising from the fact that Proposition 1 relies on a second order approximation.

2.2 Bounds on Cost of Frictions

One important assumption embedded in Proposition 1 is that the third derivative of frictions is small $\frac{d^3\delta}{dh^3} \approx 0$. What can we say about the cost of frictions if $\frac{d^3\delta}{dh^3} \not\approx 0$? Suppose that $\frac{d^3\delta}{dh^3} \not\approx 0$ but that $\frac{d^4\delta}{dh^4} \approx 0$. It is straight-forward to show (via identical fundamental theorem of calculus logic as in Proposition 1) that a third-order approximation for the cost of frictions can be computed as:

$$\frac{\delta(h^F)}{wh^*} \approx (1 - T'(wh^*)) \frac{\left(\frac{h^F - h^*}{h^*}\right)^2}{2\xi_{h^*}} + (h^*)^3 \frac{d^3\delta}{dh^3}\bigg|_{h^*} \left(\frac{h^F - h^*}{h^*}\right)^3 \bigg/ 6$$

Hence, if $\frac{d^3\delta}{dh^3} \not\approx 0$ *and* individuals face large frictions so that $\left(\frac{h^F - h^*}{h^*}\right)^3$ is big, then the approximation in Proposition 1 can incorrectly characterize the cost of frictions. While one could theoretically use estimates of $(h^*)^3 \frac{d^3\delta}{dh^3}\big|_{h^*}$ (and analogous) higher order terms to sharpen the estimates of $\delta(h^F)$, these higher order terms are functions of elasticities of elasticities, which are exceedingly difficult to estimate in practice (Kleven, 2016). Instead, we now illustrate how one can bound the cost of frictions using a revealed preference argument (see Appendix A.3 for proof):

Proposition 2. *Suppose individuals always have the choice to not work. The willingness-to-pay to remove frictions is bounded by:*

$$\frac{\delta(h^F)}{wh^*} < \frac{wh^* - T(wh^*) - [wh^F - T(wh^F)]}{wh^*} \qquad for\ h^* > h \tag{8}$$

$$\frac{\delta(h^F)}{wh^*} < \frac{wh^* - T(wh^*) + T(0)}{wh^*} \qquad for\ h^* < h \tag{9}$$

as long as $\frac{d\delta}{dh}(h) < 0\ \forall\ h < h^*$.[3]

Intuitively, for individuals who would ideally like to work more than they currently do ($h^* > h^F$), the cost of frictions is bounded above by the gain in income generated from moving from h^F to h^* because labor supply is costly. Similarly, for individuals who would ideally like to work less than they currently do ($h^* < h^F$), the cost of frictions is bounded above by the cost of moving to unemployment from h^*, which is in turn bounded above by the consumption loss of moving to unemployment from h^*. While the bounds of Proposition 2 are coarse, they are valid even when frictions are very large (and hence Proposition 1 may be inaccurate if $\frac{d^3\delta}{dh^3} \not\approx 0$).

3 Empirical Size of Adjustment Frictions

Next, we will attempt to apply Proposition 1 empirically to assess the size of adjustment frictions. There are three elements that we will need to assess the magnitude of adjustment frictions: (1) how many hours individuals actually work in the presence of adjustment frictions, h^F, (2) how many hours individuals would ideally like to work in absence of adjustment frictions h^*, and (3) the Hicksian elasticity of ideal hours h^* w.r.t. the tax rate. We will begin by discussing two datasets which have information on h^F and h^*: the U.S. National Study of the Changing Workforce and the German Socio-Economic Panel. Each of these two datasets has strengths and weaknesses relative to the other.

[3]The condition that $\frac{d\delta}{dh}(h) < 0\ \forall\ h < h^*$ just means that the WTP to remove frictions is increasing as h gets farther away from h^*. This is loosely equivalent to utility $u(c(h), h)$ being single-peaked in h. This condition always holds, for example, if $u(c, h)$ is quasi-linear in c so that $u(c, h) = c - v(h)$ with convex $v(h)$ and linear taxes as in this case:

$$c(h) + \delta(h) - v(h) = c(h^*) - v(h^*)$$

$$\frac{d\delta}{dh}(h) = v'(h) - c'(h) = v'(h) - (1 - T')w$$

$v'(h^*) - (1 - T')w = 0$ by the FOC so $v'(h) - (1 - T')w < 0\ \forall h < h^*$ by convexity of $v(\cdot)$.

3.1 Data on h and h^*

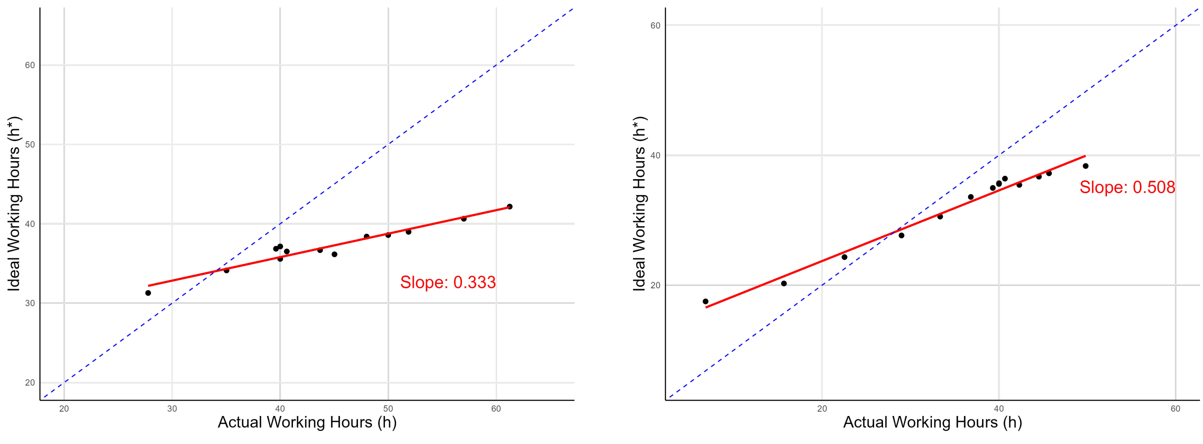
National Study of Changing Workforce (NSCW): The NSCW is a repeated cross-sectional survey conducted by The Families and Work Institute in 1992, 1997, 2002, and 2008. The survey contains a wide number of questions related to employment and demographics. For our purposes, we will use information on wages w and actual hours worked per week in the (potential) presence of frictions, h^F. Most importantly, the survey also contains a measure of ideal hours worked per week, h^*, taken from the answer to the question "If you could do what you wanted to do, ideally how many hours, in total, would you like to work each week?" In using this measure to assess adjustment frictions, it is important to consider how respondents may interpret this question. First and foremost, given the question's wording, we may be concerned that individuals do not take into account the financial cost of working less. While only 5.1% of respondents answered 0, some respondents could in theory respond by stating how many hours they think they could reasonably work while getting paid the same amount. Additionally, some individuals may respond by answering how much they would like to work in an ideal world in which their life situation was different above and beyond removing adjustment frictions; for instance, one might interpret an "ideal" world as one in which childcare was cheaper and therefore they would be able to work more. Fortunately, the NSCW contains an additional question in 2008 that asks, "Why don't you work fewer (more) hours per week?". Table **??** in Appendix **??** provides the distribution of responses to the above questions. Of the households who report that they would prefer to work less, around 48% state that the reason they do not in fact work less is because they need the money (suggesting they did not interpret the initial question on ideal hours as being about adjustment frictions).[4] Around 5% of individuals who would ideally like to work more say they do not work more due to childcare or other constraints that cannot be reasonably considered adjustment frictions. While we will provide estimates of adjustment frictions for the entire sample in Appendix **??** (assuming that everyone answered the question about ideal hours as being solely about removing adjustment frictions), our main estimates of adjustment frictions will exclude individuals who misinterpreted the initial question about ideal hours, illustrated by responding to the above question with one of the following responses "[ENTER RESPONSES HERE]".[5] Figure 1a presents a binscatter of how ideal hours varies with actual hours in the NSCW and Figure 11a in Appendix D shows the distributions of h^F

[4]However, if individuals face limited choices of hours (e.g., a discrete choice between full-time and part-time), then they may interpret this question as effectively asking why they do not work part-time (rather than why they don't work their ideal hours) at which point saying they "need the money" does not necessarily imply that their answer to the question on ideal hours ignored financial costs of working less.

[5]The remaining sample is therefore representative if we assume that the dropped individuals would face the same distribution of frictions as the rest of the sample if they correctly interpreted the question. We also illustrate an even more conservative scenario which assumes that adjustment frictions are zero (i.e., $h^* = h^F$) if individuals incorrectly interpret the question about ideal hours.

and h^* in the NSCW. Note that there is a strong relationship between actual and ideal hours; moreover workers who work fewer hours, on average, would like to work more and workers who work many hours, on average, would like to work less. [NICK: Nick, please fill in any missing hyperlinks/questions if possible.]

German Socio-Economic Panel (GSOEP): The GSOEP is a longitudinal survey of approximately 15,000 private households, representative of the German population, conducted by the *Deutsches Institut für Wirtschaftsforschung* since 1984. In addition to collecting information on hourly wages w and hours worked (possibly in the presence of frictions) h^F, we use the answer to the following question to infer h^*: "If you could choose your ideal working hours, *taking into account your income would change according to the number of hours*: How many hours would you work?" We believe that this question is phrased in a manner which enables easier interpretation of the answer as being related to adjustment frictions given that it explicitly states that income would go down with hours worked. We use the answer to this question as our measure of ideal hours worked in the absence of adjustment frictions, h^*.[6] Figure 1b presents a binscatter of how ideal hours varies with actual hours in the GSOEP and Figure 11b in Appendix D shows the distributions of h and h^* in the GSOEP. As in the NSCW, there is a strong relationship between actual and ideal hours; moreover workers who work few hours, on average, would like to work more and workers who work many hours, on average, would like to work less. [NICK: Please fill in any missing hyperlinks if possible.]



(a) NSCW

(b) GSOEP

Figure 1: Actual vs Ideal Hours

Notes: Each dot in this graph represents the average number of actual hours and the average number of ideal hours in each of the three surveys for a group of respondents. These averages are computed for 15 groups of respondents, ordered by their reported number of actual hours. The red line represents the best linear predictor of ideal hours based on actual working hours. The dashed blue line represents the 45° line.

[6] There is not a question in the GSOEP that asks why people do not work their ideal hours.

3.2 Estimating the Ideal Hours Elasticity

The final component we need to assess the size of frictions is the elasticity of ideal hours w.r.t. the tax rate. While many papers have estimated elasticities of actual hours (and income) w.r.t. tax changes, to the best of our knowledge, no paper has estimated the impact of tax changes on *ideal* hours worked, which is the relevant elasticity for understanding the cost of adjustment frictions. We now attempt to estimate this parameter using the GSOEP and variation from German tax reforms in the 1990s.[7] The variation in tax rates is discussed and depicted in Appendix B.2.

Our empirical strategy will explore how changes in ideal hours worked between pairs of years relates to tax changes between the same pairs of years for given individuals. We label the two years 1 and 2; following Feldstein (1999) and Gruber and Saez (2002), the length of time between "year 1" and "year 2" will be three years in our baseline specifications although we will show robustness to this time length. We use data on ideal hours worked from the GSOEP for years between 1991-2001, which comprises a 5 year window on either side of a large tax reform in 1996. Hence, our dataset will consist of stacked three year differences between years 1991 and 1994, 1992 and 1995, etc.[8]

The vast majority of previous papers that have analyzed behavioral responses to tax reforms have explored changes to piecewise linear tax schedules. In contrast, the German tax schedule is a non-linear (quadratic) function. Hence, we need to derive a slightly different estimating equation (relative to prior empirical work) that accounts for the non-linearities in the tax schedule. We show in Appendix B.3 how to derive the following discretized Slutsky-esque equation to recover the Hicksian ideal hours elasticity, ξ_{h^*}, for non-linear reforms:

$$\Delta \log(h^*)_i = \xi_{h^*} \Delta \log(1 - T'(wh^*))_i + \eta_{h^*} \Delta \log(wh^* - T(wh^*))_i + \nu_i \tag{10}$$

where i denotes an individual, $\Delta \log(1 - T'(wh^*))_i = [\log(1 - T_2'(w_1 h_1^*)) - \log(1 - T_1'(w_1 h_1^*))]_i$ represents the mechanical change in marginal tax rates between years 1 and 2 for individual i, $\Delta \log(wh^* - T(wh^*))_i = [\log(w_1 h_1^* - T_2(w_1 h_1^*)) - \log(w_1 h_1^* - T_1(w_1 h_1^*))]_i$ represents the mechanical change in after tax income between years 1 and 2 for individual i, and ν represents an error term. The subscripts 1 and 2 represent values in year 1 and year 2, respectively.

There are two points to discuss. First, the terms $\Delta \log(1 - T'(wh^*))_i$ and $\Delta \log(wh^* - T(wh^*))_i$ in Equation (10) represent *mechanical* changes in the keep-rate and after-tax-income holding ideal hours (and therefore ideal income) constant at year 1 level. Previous papers (e.g., Gruber and Saez (2002) or Kopczuk (2005)) that have estimated similar specifications to

[7] We cannot estimate this elasticity in the U.S. using the NSCW because the NSCW is not a panel dataset.

[8] Unfortunately, the GSOEP did not collect data on ideal hours worked in 1996 so we can not include the 1993 to 1996 and 1996 to 1999 differences in our regressions.

Equation (10) have typically included the actual changes in the keep-rate and after-tax-income and instrumented for these changes with the mechanical changes that would have ensued if there were no behavioral effects. Using these simulated instruments is of course not a problem from an econometric standpoint, but for the purposes of recovering the parameter ξ_{h^*}, theory tells us that we should *not* use the mechanical changes in taxes as simulated instruments and instead use them directly as regressors when the tax schedule is non-linear.[9]

Second, estimating Equation (10) will yield an unbiased estimate of ξ_{h^*} assuming that changes in marginal tax rates are assigned (as good as) randomly so $\nu_i \perp \Delta \log(1 - T'(wh^*))_i$. In practice, tax changes are *not* randomly assigned: the (mechanical) change experienced in both the marginal tax rate and tax liability from a reform is determined by one's income level in year 1. Hence, the key threat to identification is that $\Delta \log(h^*)_i$ is correlated with $(w_1 h_1^*)_i$: this may occur, for instance, due to differential trends in h^* across the income distribution. We employ the solution commonly adopted in previous papers (e.g., Gruber and Saez (2002)), which is to control flexibly for income in year 1. Our estimating equation is as follows where $f(w_1 h_1^*)_i$ is a flexible function of year 1 income, $Year_j$ denote base year dummies, and incomes are adjusted for inflation:

$$\Delta \log(h^*)_i = \xi_{h^*} \Delta \log(1 - T'(wh^*))_i + \eta_{h^*} \Delta \log(wh^* - T(wh^*))_i + f(w_1 h_1^*)_i + Year_j + \nu_i \tag{11}$$

Table 1 shows results from estimating Equation (11).[10] All specifications (other than Column 7) restrict to households with a single earner to abstract from joint household labor supply decisions. Robustness exercises are performed in Columns 2-9 and are detailed in the table notes. As can be seen, our estimate for ξ_{h^*} is imprecisely estimated. The baseline estimate in Column (1) is 0.13 with a 95% CI of [-0.21,0.47]. The point-estimates are fairly stable but noisy across all specifications. Importantly however, across all specifications, the 95% confidence intervals can rule out very large elasticities, with the upper limits of these confidence intervals all being less than 1; moreover, in all columns except Column (6), upper limits are less than 0.5. Thus, in the next section, we will show how the size of adjustment frictions varies for values of $\xi_{h^*} \in (0, 1]$.[11]

[9] When the initial tax schedule is linear (or piecewise linear) as in Gruber and Saez (2002) or Kopczuk (2005), then using the mechanical change in marginal tax rates as a simulated instrument or directly as a regressor is equivalent conditional on the assumption that individuals move smoothly in response to tax schedules (which is used to derive the Slutsky equation in the first place). If individuals stay within the same tax bracket then their mechanical change in marginal tax rates is equivalent to their actual change in marginal tax rates even after behavioral responses, meaning that theoretically the R^2 from the first stage should be 1. When the tax schedule is non-linear, as in our setting, these are not theoretically equivalent due to a non-zero second derivative of the tax schedule.

[10] For comparison with prior studies, we also show regressions estimated using actual hours worked and income as the dependent variable: see Table 2 and Table 3 in Appendix D.1, respectively. While both are imprecisely estimated, the point estimates are consistent with those in the literature: the actual hours (income) elasticity is 0.04 (0.26) in our baseline specification.

[11] $\xi_{h^*} > 0$ if marginal utility of consumption is positive and second order conditions hold (see Equation (25)).

Finally, while to the best of our knowledge no paper has attempted to estimate the ideal hours elasticity, several papers have estimated the earnings elasticity in absence of frictions (often referred to as the structural earnings elasticity), such as Kleven and Waseem (2013), Gelber et al. (2020), Tazhitdinova (2020). To do so, these papers have utilized bunching masses at notches and/or kinks in the income tax schedule along with functional form assumptions on the form of frictions. These papers all find that structural elasticities are larger than those estimated with frictions[12]; however, they also find fairly small structural earnings elasticities with values substantially lower than 1 (Chetty et al. (2011) also suggest this structural elasticity is less than 1). Given we expect hours elasticities to be less than earnings elasticities (both with and without frictions), this supports $\xi_{h^*} < 1$.

Table 1: Estimates of the Ideal Hours Worked Elasticity w.r.t. the Marginal Tax Rate

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Baseline	Cubic	Linear	Time Trends	<50th	Singles	All Hhs	2 Yr Diff	4 Yr Diff
ξ_{h^*}	0.16	0.10	0.14	0.15	0.13	0.38	0.13	0.04	0.13
	(0.16)	(0.15)	(0.15)	(0.16)	(0.17)	(0.24)	(0.14)	(0.11)	(0.13)
η_{h^*}	0.06	-0.02	0.19	0.09	0.23	0.27	0.14	0.11	0.13
	(0.35)	(0.33)	(0.32)	(0.36)	(0.50)	(0.52)	(0.31)	(0.24)	(0.32)
Obs.	2,830	2,845	2,845	2,830	1,690	1,599	3,940	4,067	1,952

Notes: Standard errors are clustered at the household level and are presented in parentheses. Each column presents estimates of ξ_{h^*} and η_{h^*} from regression (11) where the dependent variable is the change in log ideal hours worked for individual i, $\Delta \log h^*_{it}$. Columns (1)-(7) use 3 year differences while Columns (8) and (9) use 2 and 4 year differences, respectively. All regressions are for years 1991-2001 inclusive (note, we do not have ideal hours for 1996), include year dummies, a married dummy, and restrict to individuals for whom marital status did not change over the pair years. All regressions are weighted using household survey weights. Column (1), (4), (5), (6), (7) include a spline in lagged log household income (i.e., lagged income decile dummies interacted with lagged income); Column (4) also includes lagged income interacted with a linear time trend. Instead of a spline, Column (2) includes a cubic polynomial in lagged log household income while Column (3) includes a linear polynomial in lagged log household income. All columns except (6) and (7) restrict to single-earner households. Column (6) restricts to single individuals only. Column (7) includes both single- and dual-earner households and the dependent variable for Column (7) is wage weighted ideal hours $\Delta \log(w_i h^*_{it} + w_j h^*_{jt})$ where j represents the second earner (if there is one) and the wages are equal to wages in time period $t-3$. This specification thus estimates ξ_{z^*}, which is the relevant parameter for bounding the cost of frictions for multi-earner households in Proposition 5. Finally, Column (5) restricts to households with lagged household income below the 50th percentile.

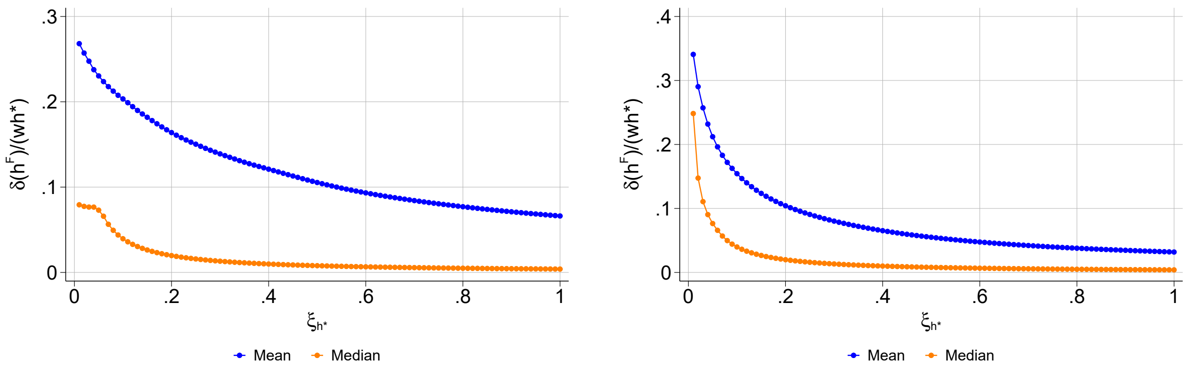
3.3 Cost of Adjustment Frictions

Next, we put all of the pieces together: we use the values of $\frac{h^F - h^*}{h^*}$ from the GSOEP and NSCW datasets combined with our range of estimates of the ideal hours elasticity from the previous section to estimate the cost of adjustment frictions from Proposition 1. As discussed in Section 2.2, for individuals with very large frictions, the second order approximation of the cost of frictions from Proposition 1 can be off; hence, we use the bounds from Proposition 2 to cap the

[12]We too find that the ideal hours elasticity is higher than the actual hours elasticity (our baseline actual hours elasticity is 0.08 - see Table 2 Column 1 - whereas our baseline ideal hours elasticity is 0.16); however, we do not have precision to say that these elasticities are statistically different.

cost of frictions derived from Proposition 1.[13] Importantly, our analysis only estimates frictions for employed individuals: all of our results require us to know the hourly wage w and this is not easily observable for unemployed individuals.[14] To begin with, we restrict our analysis to households with a single earner; we will address joint labor supply decisions in Section 4.2.1. Because our estimates of the Hicksian elasticity of ideal hours w.r.t. the marginal tax rate are imprecisely estimated, Figure 2 first illustrates how the mean and median cost of adjustment frictions (as a fraction of ideal income wh^*) varies with the elasticity of ideal hours w.r.t. the tax rate, showing the range of elasticities that are within the 95% CI of *any* of the various specifications that we consider.[15]

[WD: Nick, let's modify Figure 3 to include a black dotted vertical line for the elasticity value we get from our baseline specification and then perhaps have the whole region within our 95% CI from our baseline specification be in like light gray.]



(a) NSCW

(b) GSOEP

Figure 2: $\frac{\delta(h^F)}{wh^*}$ vs. ξ_{h^*}

Notes: This figure plots the average and median cost of adjustment frictions (as a fraction of ideal income) for single-earners for different values of the ideal hours elasticity, ξ_{h^*}, using data on ideal hours, h^* and actual hours worked, h^F, from the NSCW (left) and the GSOEP (right). For all individuals, the cost of adjustment frictions are calculated via Proposition 1; if the value from this calculation exceeds the revealed preference bounds in Proposition 2, the bounds are applied.

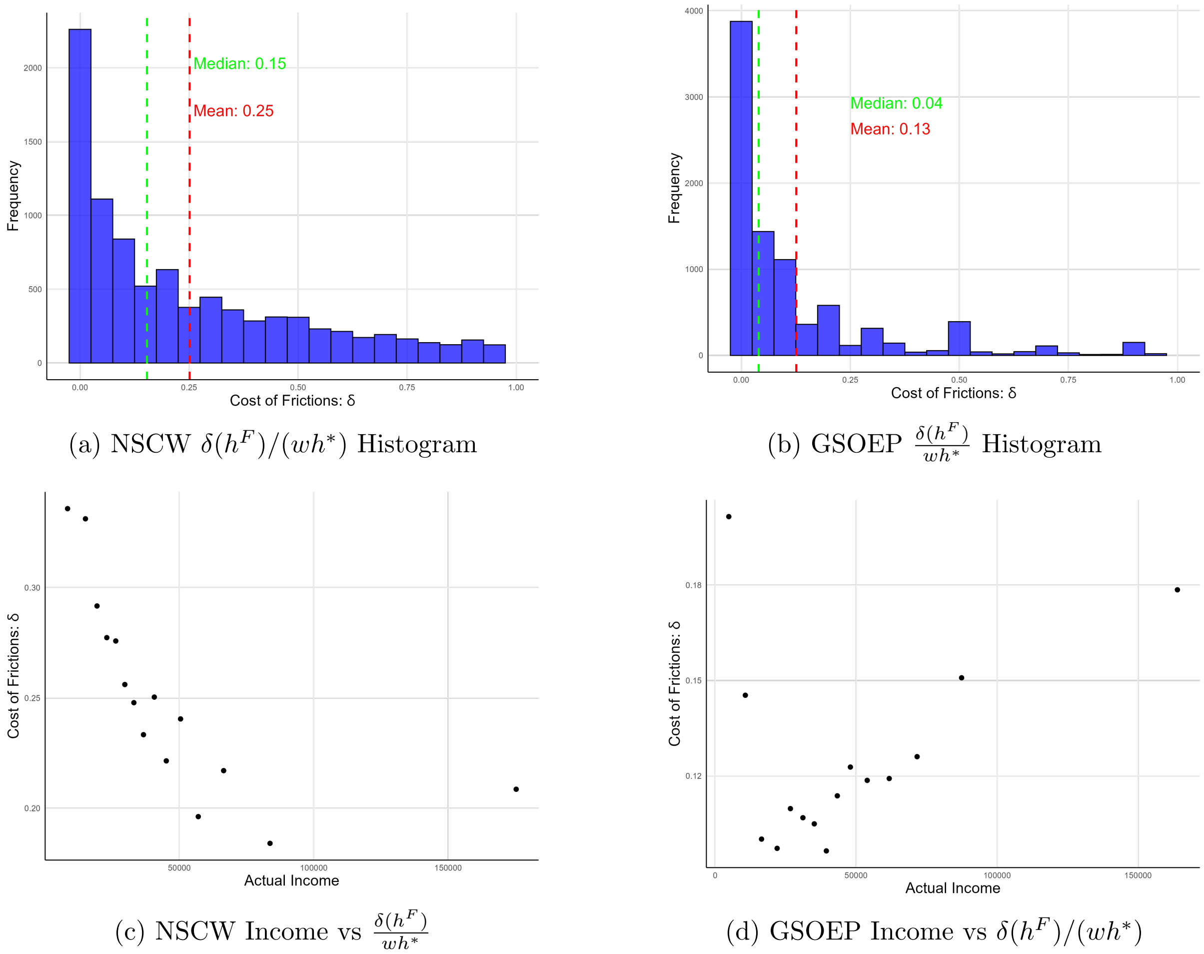
Next, we illustrate the distribution of adjustment frictions as well as how adjustment frictions vary across the income distribution. Figure 3 illustrates histograms of the cost of adjustment frictions $\frac{\delta(h^F)}{wh^*}$ for the NSCW and the GSOEP assuming the value of $\xi_{h^*} = 0.13$ from our baseline specification. First, in both the NSCW and the GSOEP we see that there is a large

[13]To explore the extent to which our second order approximation of the cost of frictions accurately gauges frictions, we illustrate in Figure 14 in Appendix D a comparison with exact calculation of frictions assuming quasi-linear isoelastic (QLIE) utility. Average frictions calculated using the second order approximation of Proposition 1 (and capped with Proposition 2) are to those calculated with the QLIE utility specification; our method appears to slightly *underestimate* frictions relative to the QLIE specification.

[14]Mas and Pallais (2019) implements a field experiment to elicit the value of employment for unemployed individuals, which can be used to recover a cost of frictions for the unemployed.

[15]Figure 12 in Appendix D illustrates that the takeaways are almost entirely unchanged if we instead consider mean and median cost of adjustment frictions as a fraction of actual income wh.

fraction of people with no adjustment frictions (i.e., those with ideal hours equal to their actual hours) and the density is mostly monotonically decreasing. In the U.S., the average (median) individual would pay XX% (YY%) of their income to remove adjustment frictions whereas in Germany the average (median) individual would pay XX% (YY%) of their income to remove adjustment frictions assuming $\xi_{h^*} = 0.13$. Figure 3 also illustrates how the size of adjustment frictions, δ, varies with (actual) income. In the U.S., we see that there is no clear relationship between the cost of adjustment frictions and income: while the average low income person would prefer to work more and the average high income person would prefer to work less, the cost of these frictions as a percentage of their income is not systematically different across the income distribution. In contrast, in Germany, we find a much starker relationship between $\delta(h^F)/(wh^*)$ and income: low income individuals face the largest frictions and then medium and high income individuals face roughly constant frictions.



(a) NSCW $\delta(h^F)/(wh^*)$ Histogram

(b) GSOEP $\frac{\delta(h^F)}{wh^*}$ Histogram

(c) NSCW Income vs $\frac{\delta(h^F)}{wh^*}$

(d) GSOEP Income vs $\delta(h^F)/(wh^*)$

Figure 3: WTP to Remove Frictions, $\frac{\delta(h^F)}{wh^*}$, as a Proportion of Income

Notes: Panels a and b show our measure of the WTP to remove frictions as a proportion of net-of-tax income, $\frac{\delta(h^F)}{wh^*}$, as described in Proposition 1. Panels c and d show binscatter plots of $\frac{\delta(h^F)}{wh^*}$ vs. actual income (in 2021 U.S. dollars). The red and green dashed vertical lines indicate the the meam and the median values of $\frac{\delta(h^F)}{wh^*}$.

4 Extensions

We now discuss how we can relax various implicit assumptions in Proposition 1 to understand the cost of frictions in more general environments.

4.1 Effort Per Hour Decisions (Endogenous Wages)

The model in Section 2.1 assumes that the only labor supply decision that individuals is how many hours to work. We show how to recover the cost of frictions in a world with two labor supply decisions: hours worked and effort per hour.[16] Hence, in this model hourly wage rate are a choice variable. This second margin of adjustment can (theoretically) allow households to mitigate the cost of hours frictions by adjusting their effort per hour.

Consider the following labor supply model in which individuals have preferences over consumption, c, hours worked, h, and effort per hour, e. Individuals have a smooth utility function $u(c, h, e)$ and face some non-linear tax schedule $T(whe)$ where where w is their effort wage:

$$c = whe - T(whe) \tag{12}$$

We are now interested in understanding the cost of frictions, $\delta(h^F, e^F)$. $\delta(h^F, e^F)$ is the amount of money required to compensate an individual for frictions which require them to work (h^F, e^F) instead of their optimal choice (h^*, e^*):

$$u(c(h^F, e^F) + \delta(h^F, e^F), h^F, e^F) = u(c(h^*, e^*), h^*, e^*) \tag{13}$$

Proposition 3 (proved in Appendix A.6) establishes that we can recover $\delta(h^F, e^F)$ in terms of h^*, h^F, and observable elasticities:

Proposition 3. *Suppose that there are no frictions in choosing effort per hour and let $\chi_{z,h}$ represent the elasticity of observed income with respect to observed hours holding ideal hours fixed. Then a second order approximation for the cost of frictions is given by:*

$$\frac{\delta(h^F, e^F)}{wh^*e^*} \approx (1 - T'(wh^*e^*))\frac{\chi_{z,h}}{2\xi_{h^*}}\left(\frac{h^F - h^*}{h^*}\right)^2 \tag{14}$$

Note, the expression for $\frac{\delta(h^F, e^F)}{wh^*e^*}$ from Proposition 3 is strikingly similar to the expression for $\frac{\delta(h^F)}{wh^*}$ from Proposition 1: the only difference is that there is one extra term which captures the impacts of effort decisions on income. Note that $\chi_{z,h}$ is equal to 1 if there is no effort decision so that Proposition 3 actually nests Proposition 1 as a special case. The core intuition is that, under the assumption that effort is chosen without frictions, we can infer how effort would change in absence of hours frictions via the parameter $\chi_{z,h}$ which captures how effort

[16] Examples of higher effort per hour (which lead to higher income) might be an Uber driver taking fewer breaks and thereby completing more trips per hour or a salaried worker increasing their effort to get a higher year-end bonus or promotion.

changes with exogenous changes in hours worked.

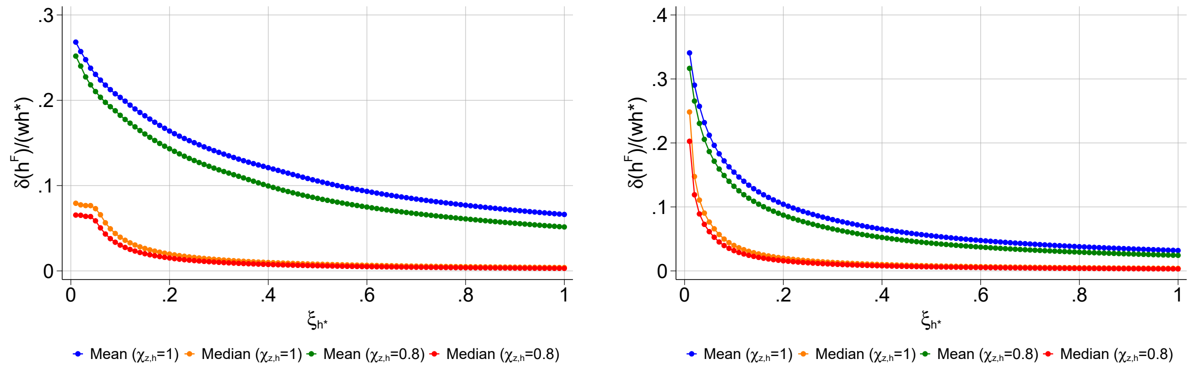
4.1.1 Calculating Cost of Frictions with Endogenous Wages

Using data from the NSCW and GSOEP, we explore how our estimates of the cost of frictions change if we incorporate endogenous wages as in Proposition 3. We repeat the exercise from Section 3.3 illustrating how the average and median cost of frictions varies with ξ_{h^*}. First, we need an estimate of $\chi_{z,h} = \frac{\partial \log(whe(h))}{\partial \log(h)}\big|_{h^*}$. This parameter reflects how income varies if hours worked changes *exogenously* due to frictions. Fortunately, a number of papers have estimated this parameter. Pencavel (2015) uses variation in hours worked for munitions workers in World War II to estimate the relationship between productivity and hours worked, estimating an average value of $\chi_{z,h} = 0.8$ (see their Table 3) and Collewet and Sauermann (2017) use exogenous variation in hours worked in call centers to estimate a value of $\chi_{z,h} \in [0.8, 0.9]$ (see their Table 3 which estimates $\chi_{z,h} - 1$ to be between 0.11 and 0.18 depending on specification).

Putting the pieces together, we form estimates for the cost of frictions in the NSCW and GSOEP from Proposition 3 setting $\chi_{z,h} = 0.8$. To limit inaccuracies from the second order approximations from Proposition 3 when $\left(\frac{h^F - h^*}{h^*}\right)$ is large, we construct bounds for the value of frictions with endogenous wages analogously to Proposition 2: see Proposition 9 in Appendix B.1. Ultimately, accounting for effort decisions lowers mean frictions in the population by around 11% in the NSCW and 15% in the GSOEP (when $\xi_{h^*} = 0.13$); see Figure 4 in Appendix D.[17] The takeaway is that even if households can reduce the cost of hours frictions by substituting to supplying more effort per hour, the size of this effect empirically (measured by $\chi_{z,h}$) is not large enough to drastically reduce our estimated cost of adjustment frictions.

[WD: Eventually, this figure will go to Appendix; I've left it here for consistency of the numbering that I sent to Nick.]

[17] Note, the value of $\chi_{z,h}$ does not change the estimated cost of frictions exactly proportionally (as to be expected from Proposition 3) because the bounds used to cap the estimated size of frictions in Proposition 9 are related in a non-linear way to the size of $\chi_{z,h}$.



(a) NSCW (b) GSOEP

Figure 4: $\frac{\delta(h^F)}{wh^*}$ vs. ξ_{h^*}

Notes: This figure plots the average and median cost of adjustment frictions (as a fraction of ideal income) for single-earners for different values of the ideal hours elasticity, ξ_{h^*}, using data on ideal hours, h^* and actual hours worked, h^F, from the NSCW (left) and the GSOEP (right). For all individuals, the cost of adjustment frictions are calculated via Proposition 3 assuming a value of $\chi_{z,h} = 1$ (see blue and orange lines; these are the same as Figure 2) and $\chi_{z,h} = 0.8$ (see green and red lines); if the value from this calculation exceeds the revealed preference bounds in Proposition 9, the bounds are applied.

4.2 Many Hours Worked Choices

The model in Section 2.1 assumed that there was only one hours worked decision. However, in reality, there are situations in which households make many labor supply decisions. For instance, one may consider (1) households where both spouses work, (2) households where a single individual works multiple jobs, or (3) households with a single individual that chooses how much to work throughout different portions of the day/week/year and has different disutility of work done during different periods of time. Appendix A.4 proves the following Proposition, showing that one can recover an estimate of the cost of frictions with many labor supply choices as long as one can observe the Slutsky matrix of own and cross-price elasticities of each ideal hours choice with respect to the after-tax wage:

Proposition 4. *One can construct a second order approximation for the cost of frictions with many labor supply choices via the Slutsky matrix of own-price and cross-price compensated elasticities:*

$$\begin{bmatrix} \frac{\partial h_1^*}{\partial((1-T_1')w_1)} \frac{(1-T_1')w_1}{h_1^*}|_C & \frac{\partial h_2^*}{\partial((1-T_1')w_1)} \frac{(1-T_1')w_1}{h_2^*}|_C & \cdots \\ \vdots & \ddots & \vdots \\ \frac{\partial h_1^*}{\partial((1-T_m')w_m)} \frac{(1-T_m')w_m}{h_1^*}|_C & \cdots & \frac{\partial h_m^*}{\partial((1-T_m')w_m)} \frac{(1-T_m')w_m}{h_m^*}|_C \end{bmatrix}$$

The key takeaway from Proposition 4 is that in order to recover the cost of frictions with many choices of hours worked, one needs the entire Slutsky matrix of ideal hours elasticities with respect to the (after-tax) wage rate of the given hours choice as well as with respect to the (after-tax) wage rate of all of the other hours choices. In some situations this may be feasible. For instance, in a model of spousal labor supply choices, one would need exogenous variation in (after-tax) wage rates separately for each spouse; this could be estimated with a tax reform in a country where couples are taxed individually rather than jointly. In other situations, however, recovering the full Slutsky matrix is almost certainly impossible. For instance, for an individual choosing how many hours to work on different days of the week, one would require separate variation in after-tax wage rates for each day of the week, which seems difficult to find quasi-experimentally.

Arguably, a more useful result is the following Proposition which establishes that the logic of Proposition 1 can be used to provide a *lower bound* for the cost of frictions for households with multiple hours worked decisions using only a single elasticity of ideal income with respect to the tax rate. We denote the vector of labor supply decisions as $\mathbf{h}$ and the cost of frictions as $\delta(\mathbf{h}^F)$ and assume that taxes are a function solely of total income $z = \sum_i w_i h_i$ where $z^F \equiv \sum_i w_i h_i^F$ denotes observed income chosen with frictions and $z^* \equiv \sum_i w_i h_i^*$ denotes ideal income:

Proposition 5. *Suppose that the third derivatives of $\delta(\mathbf{h}) \approx 0$ and that utility is additively separable between c and $\mathbf{h}$. A lower bound for the cost of frictions expressed as a fraction of optimal income is given by:*

$$\frac{\delta(\mathbf{h}^F)}{z^*} \geq \left(1 - T'\left(z^*\right)\right) \frac{\left(\frac{z^F - z^*}{z^*}\right)^2}{2\xi_{z^*}}$$

where ξ_{z^*} represents the Hicksian elasticity of z^* with respect to the marginal tax rate.

The intuition behind Proposition 5 (see the proof in Appendix A.5) is that the Hicksian elasticity of frictionless income z^* informs the curvature of utility with respect to ideal income *assuming households optimize their labor supply decisions conditional on earning a given income* (that is, assuming each h_i is chosen optimally subject to earning a total amount $\sum_i w_i h_i$). Given that households who earn some amount z^F may *not* be optimizing each individual labor supply decision conditional on their earnings level, this forms a lower bound for the cost of frictions. In other words, Proposition 5 captures the cost of frictions which cause a household to deviate from frictionless income but does not capture the cost of frictions which lead to a sub-optimal allocation of hours worked given a total income level.

4.2.1 Calculating Cost of Frictions for Dual-Earner Households

We now apply the results from the previous section to recover the cost of frictions for households with two earners (recall that our results in Section 3 were exclusively for households with a single earner). The NSCW only surveyed a single member of each household and therefore did not collect data on the ideal hours of work for both spouses; hence, all of the analysis in this section will only use the GSOEP. First, we implement Proposition 5 to recover a lower bound for the size of frictions for couples; Figure 5 in Appendix D shows how the lower bound estimate for the average and median size of frictions for two earner households varies with the parameter ξ_{z^*}.

When $\xi_{z^*} = 0.13$ (our point estimate of the ideal income elasticity from Column (7) of Table 1), the mean (median) WTP to remove adjustment frictions is 9% (3%) of pre-tax income.

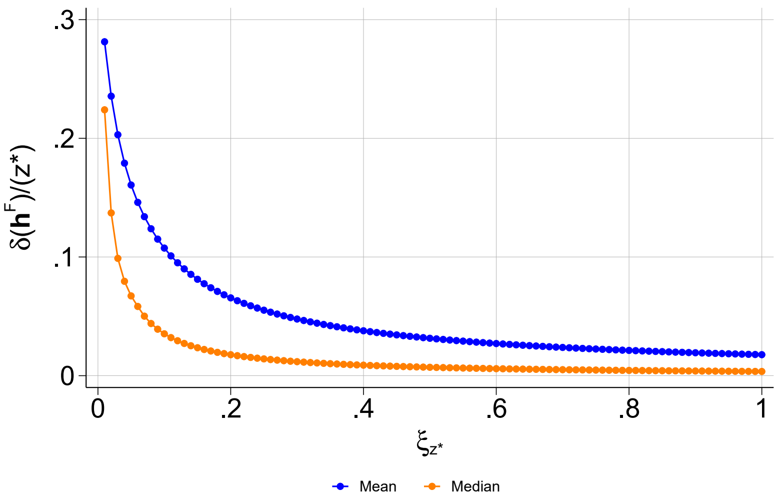


Figure 5: Lower Bound on $\frac{\delta(\mathbf{h}^F)}{z^*}$ for Dual-Earner Households

Notes: This figure plots a lower bound for the average and median cost of adjustment frictions (as a fraction of ideal income) for dual-earner households for different values of the ideal income elasticity, ξ_{z^*}, using data on ideal hours, h^*, and actual hours worked, h^F, from the GSOEP. For all households, the lower bound for the cost of adjustment frictions are calculated via Proposition 5; if this household lower bound exceeds that of the combined individual revealed preference bounds from Proposition 2, the household lower bound is replaced with the combined individual revealed preference bounds.

4.3 Dynamic Extension

In this section, we show how Proposition 1 extends to a dynamic setting. We consider a recursive formulation with two state variables: the wage in period t, w_t, and the individual's wealth level in period t, κ_t. There are two choice variables in each time period: how many hours to work h_t and how much to save σ_t subject to an interest rate r_t. Furthermore, we account for "career concerns" by allowing w_{t+1} to be a function of h_t. There may also be randomness in future wages or interest rates or preferences in the future. The individual's problem can be written as:

$$\max_{h_t} \left\{ \max_{\sigma_t} u(w_t h_t - T_t(w_t h_t) - \sigma_t, h_t) + \mathbb{E}[V(w_{t+1}(h_t), \kappa_t + r_t \sigma_t)] \right\} \tag{15}$$

Let us suppose that either there are no frictions in σ_t *or* that σ_t is at a corner solution and does not change with h_t (e.g., due to borrowing constraints). Let $\sigma_t^*(h_t)$ denote the optimal level of savings for a given level of hours worked. Let us define $\delta(h_t^F)$ as the WTP to remove frictions in hours worked in time period t:

$$\begin{aligned} & u(w_t h_t^F - T_t(w_t h_t^F) - \sigma_t^*(h_t^F) + \delta(h_t^F), h_t^F) + \mathbb{E}[V(w_{t+1}(h_t^F), \kappa_t + \sigma_t^*(h_t^F))] \\ &= u(w_t h_t^* - T_t(w_t h_t^*) - \sigma_t^*(h_t^*), h_t^*) + \mathbb{E}[V(w_{t+1}(h_t^*), \kappa_t + \sigma_t^*(h_t^*))] \end{aligned} \tag{16}$$

In Appendix A.7 we prove the following Proposition which establishes that with the properly defined elasticity, Proposition 1 still holds:

Proposition 6. *Suppose that* $\frac{d^3\delta}{dh_t^3} \approx 0$. *The WTP to remove frictions expressed as a fraction of optimal income* $w_t h_t^*$ *can be approximated as:*

$$\frac{\delta(h_t^F)}{w_t h_t^*} \approx (1 - T_t'(w_t h_t^*)) \frac{\left(\frac{h_t^F - h_t^*}{h_t^*}\right)^2}{2\xi_{h_t^*}} \tag{17}$$

where $\xi_{h_t^*}$ *is the compensated elasticity of ideal hours in time period* t *w.r.t. the tax rate in time period* t *holding all future tax rates constant.*

There are a few things to note regarding Proposition 6. First, we do not assume anything about the continuation utility function $V(w_{t+1}, \kappa_t + \sigma_t)$. In particular, there may be frictions in future choices of labor supply or future savings; hence, Proposition 6 recovers the cost of hours frictions in time period t conditional on future frictions. Second, the elasticity required, $\xi_{h_t^*}$, captures the compensated change in ideal hours in time period t for a change in marginal tax rates *in period* t, holding constant *all future tax rates.* Hence, this elasticity is potentially difficult to estimate empirically not only because one requires data on ideal hours but also because one requires *temporary* variation in tax rates. Finally, let us discuss how Proposition 6 impacts the conclusions from Section 3. The relationships between the size of frictions and the elasticity in Figure 2 still hold (if we interpret the x-axis as now representing $\xi_{h_t^*}$); however, to the extent that agents are not myopic, the elasticities estimated in Section 3.2 (which are estimated using permanent rather than temporary tax changes) are no longer the relevant elasticities for understanding the cost of frictions because these elasticities conflate hours changes today that result due to tax changes today with increases in hours worked today that result from tax changes in the future. The elasticities estimated in Section 3.2 are equal to $\xi_{h_t^*}$ used in Proposition 6 if agents do not consider future tax changes when making labor supply decisions today.

5 Tax Misperceptions

Next, we explore how misperceptions impact our estimates of the cost of frictions. There are two key takeaways: (1) in the presence of misperceptions, the cost of adjustment friction estimates from Section 3 should be interpreted as the cost of frictions *conditional on misperceptions*, and (2) we can incorporate misperceptions of the tax schedule into a total cost of frictions inclusive of adjustment frictions - this exercise suggests that the costs of adjustment frictions are much larger than tax misperceptions and that the cumulative cost of frictions is large *regardless* of the value of the Hicksian elasticity.

5.1 Adjustment Frictions with Misperceptions

First, note that Proposition 1 does not require that the tax schedule in question is the true tax schedule. If the true tax schedule is given by $T(wh)$ but agents misperceive the tax schedule as $\hat{T}(wh)$, then we can replace T with $\hat{T}$ in Proposition 1 and interpret h^* as ideal hours conditional on misperceptions to recover the WTP to remove adjustment frictions conditional on misperceptions of the tax schedule.[18] Hence, our results from Section 3 should all be interpreted as how much individuals would be willing to pay to remove frictions conditional on existing misperceptions of the tax schedule.[19] The caveat is that when individuals misperceive the tax schedule, the relevant elasticity for Proposition 1 is now the Hicksian elasticity of ideal hours with respect to the *perceived* marginal tax rate holding the *perceived* tax level constant. Hence, unless individuals misperceive the existing tax schedule yet correctly perceive tax reforms, the elasticity estimates from Section 3.2 will no longer be relevant (but the relationships between the size of frictions and this unknown elasticity in Section 3 are otherwise still valid if interpreted as costs conditional on misperceptions).

5.2 Cost of Adjustment Frictions and Misperceptions

But what if we want to estimate the cumulative ex-post cost of both adjustment frictions and tax misperceptions? Going back to the baseline model of Proposition 1, suppose that an individual works h^F hours given their misperceptions of the tax schedule and their adjustment frictions. Let h^* denote ideal hours conditional on tax misperceptions (i.e., without any adjustment frictions). And let us denote h^{**} as the ideal hours this individual would like to work if they faced neither adjustment frictions nor tax misperceptions. Proposition 1 can be applied to recover the cumulative cost of frictions with the caveat that the relevant elasticity is now $\xi_{h^{**}}$, the Hicksian elasticity of h^{**} w.r.t. the marginal tax rate if the tax schedule was correctly perceived:

$$\frac{\delta(h^F)}{wh^{**}} \approx (1 - T'(wh^{**}))\frac{\left(\frac{h^F - h^{**}}{h^{**}}\right)^2}{2\xi_{h^{**}}} \tag{18}$$

Let us first discuss how we might recover h^{**} so as to gauge the cumulative cost of frictions. First, let us assume that we have an estimate of h^*, which represents how many hours an individual would like to work in absence of adjustment frictions but conditional on misperceptions (this is presumably what one recovers from a survey such as the NSCW or GSOEP). So how can we recover h^{**} from an estimate of h^*? Using the discrete approximation to the Slutsky-esque in Equation (10), the following characterizes (approximately) how ideal hours worked changes with the perceived tax schedule:

$$\begin{aligned} &\log(h^*) - \log(h^{**}) \approx \xi_{h^{**}}[\log(1 - \hat{T}'(wh^{**})) - \log(1 - T'(wh^{**}))] \\ &+ \eta_{h^{**}}[\log(wh^{**} - \hat{T}(wh^{**})) - \log(wh^{**} - T(wh^{**}))] \end{aligned} \tag{19}$$

[18] In the terminology of the behavioral economics literature, we will recover the ex ante WTP to remove adjustment frictions under the decision utility function $u(wh - \hat{T}(wh), h)$ rather than the ex post WTP to remove adjustment frictions under the experience utility function $u(wh - T(wh), h)$.

[19] Moreover, to the extent that individuals misperceive other elements of the environment, such as their utility function or their wage, the results from Section 3 should be interpreted as capturing the WTP to remove frictions conditional on these frictions as well.

where $\xi_{h^{**}}$ is the elasticity of ideal hours worked w.r.t. the marginal tax rate evaluated at h^{**} and $\eta_{h^{**}}$ is the elasticity of ideal hours with respect to the after-tax income evaluated at h^{**}. Combining Equations (18) and (19) yields:

Proposition 7. *Suppose that* $\frac{d^3\delta}{dh^3} \approx 0$. *The WTP to remove frictions expressed as a fraction of optimal income* wh^{**} *can be approximated as:*

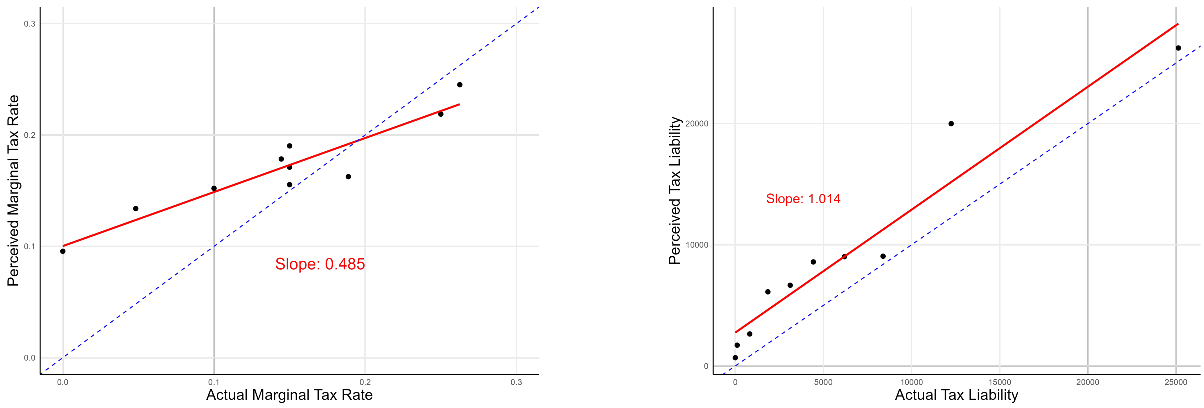
$$\frac{\delta(h^F)}{wh^{**}} \approx (1 - T'(wh^{**})) \frac{\left(\frac{h^F - h^* \left[\frac{1-T'(wh^{**})}{1-\hat{T}'(wh^{**})}\right]^{\xi_{h^{**}}} \left[\frac{wh^F - T(wh^{**})}{wh^F - \hat{T}(wh^{**})}\right]^{\eta_{h^{**}}}}{h^* \left[\frac{1-T'(wh^{**})}{1-\hat{T}'(wh^{**})}\right]^{\xi_{h^{**}}} \left[\frac{wh^F - T(wh^{**})}{wh^F - \hat{T}(wh^{**})}\right]^{\eta_{h^{**}}}} \right)^2}{2\xi_{h^{**}}} \tag{20}$$

The key takeaway from Proposition 7 is that when there are both adjustment frictions and misperceptions, the relationship between the cost of frictions and $\xi_{h^{**}}$ is now theoretically ambiguous. Intuitively, larger values of $\xi_{h^{**}}$ still imply that there is less curvature in utility (so that the cost of frictions is smaller) but they now also imply that individuals may have larger deviations from their optimal hours worked due to misperceptions (because people are more responsive to tax misperceptions). Hence, as $\xi_{h^{**}}$ increases, the cost of adjustment frictions goes down but the cost of misperceptions may increase.

5.3 Data on Misperceptions

In order to gauge the impact of tax misperceptions via Proposition 7, we need data on the size of misperceptions in both the marginal tax rate and tax liability. For this purpose we use estimates of tax misperceptions from Rees-Jones and Taubinsky (2020) (RJT). RJT conducted an incentivized survey with a representative sample of 4,197 U.S. taxpayers to elicit perceptions of the U.S. income tax schedule. The survey asks respondents to assess the tax liability at their own income and at 16 other income levels.[20] Hence, the RJT survey provides data on individuals' perceptions of their own perceived tax liability $\hat{T}$ and allows us to infer their perceptions about marginal tax rates from their misperceptions about how perceived tax liability is changing with income. Precisely, we will use their perceived tax liability at the farthest income level that's in the same tax bracket to infer their perceived marginal tax rate (see Appendix B.4 for more discussion). Figure 6 plots binscatters of perceived marginal tax rates vs. actual marginal tax rates and perceived tax liability vs. actual tax liability. There are strong positive correlations between perceptions and actuality in both cases. Individuals with lower marginal rates tend to over-estimate their marginal rates while individuals with higher marginal rates tend to under-estimate their marginal rates. Most individuals seem to over-estimate their tax liability.

[20] Respondents are asked to assess the tax liability for a fictional taxpayer "Fred" with income level z_F, but otherwise identical to themselves. Participants were informed they may receive a monetary compensation if their answers are within \$100 of the true tax liability.



(a) Actual vs Perceived Mg Tax Rates

(b) Actual vs Perceived Tax Liability

Figure 6: Actual and Perceived Taxes

Notes: Each bin indicates the average actual marginal tax rate (actual tax liability) against the average perceived marginal tax rate (perceived tax liability) in Panel a (b). These averages are computed for 10 groups of respondents ordered by their actual marginal tax rates (actual tax liability). The red line represents the best linear predictor of perceived marginal tax rates (perceived tax liability) given the actual marginal tax rate (actual tax liability). The dashed blue line represents the 45-degree line.

5.4 Estimating Cumulative Cost of Frictions

To understand the cumulative cost of adjustment frictions and tax misperceptions via Proposition 7 we need data on both h^F and h^* along with the size of tax misperceptions. Given that we do not have data on all of these objects in the same dataset, we "join" the NSCW data and the RJT data in order to roughly gauge the combined cost of adjustment frictions and tax misperceptions. We now describe this data "join".

We start with the NSCW dataset from 2008 which has data on h^F and h^* (recall h^* represents ideal income without adjustment frictions but with misperceptions). For each individual, we calculate their rank in the income distribution from 1 to 100. Similarly, in the RJT data, which has information on $\hat{T}'$ and $\hat{T}$, we calculate each individuals rank in the income distribution from 1 to 100. Next, we perform a "join" of these two datasets, matching all pairs of observations with the same income percentile to create a synthetic dataset of both adjustment frictions and tax misperceptions. There are two modestly heroic assumptions that we implicitly make when performing this exercise: (1) the distribution of tax misperceptions conditional on income rank has not changed from the 2008 NSCW to the RJT survey conducted in YYYY, and (2) the size of tax misperceptions is independent of the size of adjustment frictions *conditional on income rank*. Under these two assumptions, the distribution of tax misperceptions faced by individuals at each income rank in the NSCW is exactly the distribution of tax misperceptions in the RJT dataset; in this case, the joined dataset accurately captures the joint distribution of adjustment frictions and tax misperceptions.

With this joined dataset, we can apply Proposition 7 to gauge the cumulative cost of adjustment frictions and tax misperceptions (applying Proposition 2 with h^* replaced by h^{**} to cap second order approximations from Proposition 7). Figure 7 displays how the mean and median size of frictions from this exercise vary with the size of the unknown parameter $\xi_{h^{**}}$; for comparison, Figure 7 also shows the cost of frictions assuming there are no misperceptions as in Figure 2a.[21] [22] There are two key takeaways. First, for small values of $\xi_{h^{**}}$, the cumulative cost of adjustment frictions and misperceptions is not overly different from the cost of adjustment frictions on their own ignoring misperceptions. Intuitively, when $\xi_{h^{**}}$ is small, individuals do not change their behavior substantially in response to misperceptions (see Equation (19)) so that the cost of misperceptions is small. Overall, adjustment frictions seem to be substantially more costly than tax misperceptions: we perform a decomposition exercise in Appendix B.5 which shows that adjustment frictions contribute 90% (80%) of the cumulative cost of frictions even when $\xi_{h^{**}} = 0.6$ ($\xi_{h^{**}} = 1$). The second takeaway, however, is that the cost of adjustment frictions and the cost of misperceptions move in opposite directions with $\xi_{h^{**}}$. Loosely, as $\xi_{h^{**}}$ becomes larger, adjustment frictions get smaller (because there is less curvature in utility) but tax misperceptions lead to larger and larger deviations from ideal hours worked, creating larger costs of tax misperceptions. Hence, the cumulative cost of frictions appears large *no matter the value of* $\xi_{h^{**}}$.[23]

[21] We assume that there are no income effects so that $\eta = 0$; Figure 13 in Appendix D shows a corresponding plot when $\eta = -0.5$. When $\eta \neq 0$, the cost of frictions is slightly larger (regardless of whether $\eta < 0$ or $\eta > 0$).

[22] To simplify computations, we assume that individuals' misperceptions of their marginal tax rate at their ideal income level are equal to their misperceptions at their actual income level so that $\log(1 - \hat{T}'(wh^{**})) - \log(1 - T'(wh^{**})) = \log(1 - \hat{T}'(wh)) - \log(1 - T'(wh))$ and $\log(wh^{**} - \hat{T}(wh^{**})) - \log(wh^{**} - T(wh^{**})) = \log(wh - \hat{T}(wh)) - \log(wh - T(wh))$. In other words, we assume wh^{**} is in the same perceived tax bracket as wh^F.

[23] While we do not have individual level data on the size of misperceptions in Germany, Appendix B.6 explores the cumulative cost of frictions in the GSOEP under the assumption that all individuals misperceive their average tax rate as their marginal tax rate as suggested in De Bartolome (1995) or Rees-Jones and Taubinsky (2020).

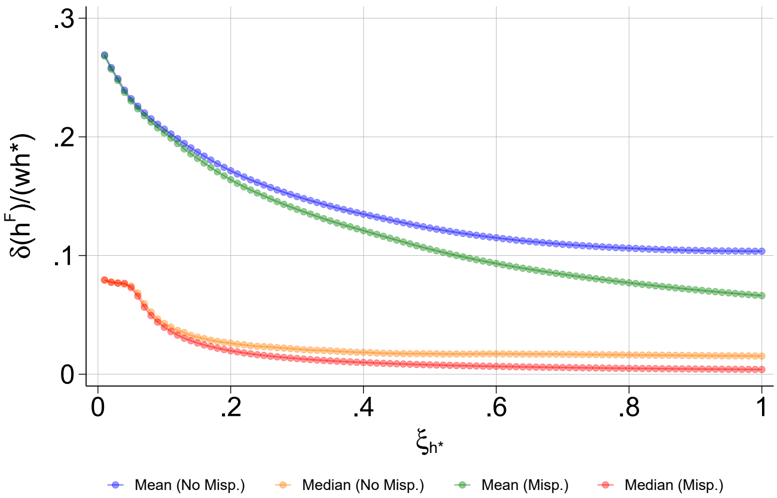


Figure 7: Cumulative Cost of Frictions

Notes: This figure illustrates the cumulative cost of frictions as a fraction of ideal income wh^* accounting for both adjustment frictions and misperceptions. Estimates of the cost of frictions are computed via Propositions 7 and 2. For comparison, we also display the cost of adjustment frictions without misperceptions (or, equivalently, the cost of adjustment frictions conditional on misperceptions) computed via Propositions 1 and 2.

6 Relationship to Chetty (2012)

The methodological contribution of the present paper is to illustrate how to use data on ideal and actual hours worked along with the structural elasticity of ideal hours worked to recover the WTP to remove frictions. In contrast, Chetty (2012) illustrates how to use reduced form elasticities (estimated in the presence of frictions) as well as an assumption about the maximum size of frictions to place bounds on the value of the structural elasticity. We now briefly discuss what we can learn about structural elasticities by combining these two approaches.

At a high level, Chetty (2012) shows that one can place bounds on the structural elasticity $\xi_{h^{**}}$ using the estimated reduced form value of $\hat{\xi}_{h^F} = \frac{\partial h^F}{\partial(1-T')}\frac{1-T'}{h^F}|_u$ and the maximum value of frictions agents are ever subject to $\bar{\delta}$ (see his Proposition 1):

$$\xi_{h^{**}} \in [\xi^L_{h^{**}}(\hat{\xi}_{h^F}, \bar{\delta}), \xi^U_{h^{**}}(\hat{\xi}_{h^F}, \bar{\delta})]$$

where $\xi^L_{h^{**}}(\hat{\xi}_{h^F}, \bar{\delta})$ and $\xi^U_{h^{**}}(\hat{\xi}_{h^F}, \bar{\delta})$ are implicitly defined by:

$$\hat{\xi}_{h^F} = \xi^L_{h^{**}}(\hat{\xi}_{h^F}, \bar{\delta}) + 2\frac{(2\xi^L_{h^{**}}(\hat{\xi}_{h^F}, \bar{\delta})\bar{\delta})^{1/2}}{\Delta\log(1-T')}$$

$$\hat{\xi}_{h^F} = \xi^U_{h^{**}}(\hat{\xi}_{h^F}, \bar{\delta}) - 2\frac{(2\xi^U_{h^{**}}(\hat{\xi}_{h^F}, \bar{\delta})\bar{\delta})^{1/2}}{\Delta\log(1-T')}$$

where $\Delta\log(1-T')$ is the variation in tax rates used to identify $\hat{\xi}_{h^F}$.

Our results show how to construct an estimate of $\bar{\delta}$ from data on $\frac{h^F-h^{**}}{h^{**}}$ across the population and the value of $\xi_{h^{**}}$. Taken together, this leads to an implicit formulation for the set identification of $\xi_{h^{**}}$ using (1) an estimate $\hat{\xi}_{h^F}$ and (2) data on the size of hours frictions, $\frac{h^F - h^{**}}{h^{**}}$. Letting $\bar{\delta}\left(\xi_{h^{**}}; \left\{\frac{h^F - h^{**}}{h^{**}}\right\}\right)$ represent the maximum size of frictions given an elasticity $\xi_{h^{**}}$ and a set of data on $\left\{\frac{h^F - h^{**}}{h^{**}}\right\}$ across the population:

Proposition 8.

$$\xi_{h^{**}} \in \left[\xi^L_{h^{**}}\left(\hat{\xi}_{h^F}, \bar{\delta}\left(\xi_{h^{**}}; \left\{\frac{h^F - h^{**}}{h^{**}}\right\}\right)\right), \xi^U_{h^{**}}\left(\hat{\xi}_{h^F}, \bar{\delta}\left(\xi_{h^{**}}; \left\{\frac{h^F - h^{**}}{h^{**}}\right\}\right)\right)\right] \quad (21)$$

Essentially, Proposition 8 extends Proposition 1 of Chetty (2012) to endogenize the value of $\bar{\delta}$.

To illustrate what we can learn about the structural labor supply elasticity $\xi_{h^{**}}$ from Equation (21), suppose we want to know what values of $\xi_{h^{**}}$ are consistent with a given value of $\hat{\xi}_{h^F}$. We can perform the following procedure. Assume a value of $\xi_{h^{**}}$ and then use Proposition 7 along with Proposition 2 (replacing h^* with h^{**}) to construct estimates for the size of $\delta(h^F)$ for each individual and then take the 99th percentile (to remove extreme outliers) of $\delta(h^F)$ in the population to construct a measure of $\bar{\delta}$. Given this value of $\bar{\delta}$ and $\hat{\xi}_{h^F}$, see if Equation (21) is satisfied. If so, then $\xi_{h^{**}}$ is consistent with $\hat{\xi}_{h^F}$. Iterate over all values of $\xi_{h^{**}}$ to identify all such $\xi_{h^{**}}$ consistent with $\hat{\xi}_{h^F}$. We illustrate the results of this procedure in Figure **??** which shows the set of $\xi_{h^{**}}$ consistent with various values of $\hat{\xi}_{h^F}$ given estimates of the costs of frictions from Propositions 7 and 2:

[WD: Let's also do this with say the average value of δ and show its still enormous. Nick please add these figures when you can.]

7 Conclusion

This paper has developed a framework to estimate the cost of labor market frictions using data on ideal and actual hours of work along with the elasticity of ideal hours worked w.r.t. the tax rate. We have also shown how to extend this framework to account for endogenous wages, multiple labor supply decisions, dynamic decision making environments, and misperceptions of the tax schedule. Empirically, the core finding is that the cost of hours frictions is large: individuals would be willing to pay at least 10% of their income on average to remove frictions in hours worked. Moving forward, we think that this research has direct applicability for the desirability of policies aimed at reducing frictions. For instance, our findings suggest that increasing the availability of gig work, reducing fixed costs of hiring employees (which generate demand side frictions), and jobs guarantees may all be beneficial to reduce the substantial welfare costs of frictions. More work investigating how these sorts of policies mitigate the cost of frictions is certainly warranted.

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A Appendix: Proofs

A.1 Proof of Equation (5)

Proof. Applying the implicit function theorem to Equation (3):

$$\frac{d^2\delta}{dh^2}\Big|_{h^*} u_c(c(h^*),h^*) + \frac{du_c(c(h),h)}{dh}\Big|_{h^*}\frac{d\delta}{dh}\Big|_{h^*} + \frac{d\left[u_c(c(h)+\delta(h),h)w(1-T'(wh)) + u_h(c(h),h)\right]}{dh}\Big|_{h^*} = 0$$

Recognizing that $\frac{d\delta}{dh}|_{h^*} = 0$ and

$$\begin{aligned} &\frac{du_c(c(h),h)w(1-T'(wh)) + u_h(c(h),h)}{dh}\Big|_{h^*} \\ &= u_{cc}(c(h^*),h^*)w^2(1-T'(wh^*))^2 - u_c(c(h^*),h^*)w^2T''(wh^*) \\ &+ 2u_{ch}(c(h^*),h^*)w(1-T'(wh^*)) + u_{hh}(c(h^*),h^*) \end{aligned} \tag{22}$$

yields Equation (5) (omitting function arguments, noting everything is evaluated at h^*). □

A.2 Proof of Lemma 1

We first need to define the Hicksian elasticity when faced with a non-linear tax schedule as in Jacquet et al. (2013). First, consider how agents would respond if they faced no frictions to a non-linear tax change from $T(wh)$ to $T(wh) + \mu\tau(wh)$ (for some smooth perturbation direction $\tau(wh)$). For an individual where $T'(wh^*) + \mu\tau'(wh^*)$ exists at their optimal income level h^*, their FOC w.r.t. h under the perturbed tax schedule reads:

$$u_c(wh^* - T(wh^*) - \mu\tau(wh^*), h^*)w(1 - T'(wh^*) - \mu\tau'(wh^*)) + u_h(wh^* - T(wh^*) - \mu\tau(wh^*), h^*) = 0 \tag{23}$$

For individuals with a unique optimal h^* whose second order condition holds strictly, we can use the implicit function theorem to describe how their optimal income changes with μ for any $\tau(z)$. Differentiating Equation (23) with respect to μ and evaluating at $\mu = 0$ yields:

$$\begin{aligned} \frac{\partial h^*}{\partial \mu} &= \frac{u_c w \tau'(z^*) + u_{cc} w(1 - T')\tau(z^*) + u_{ch}\tau(z^*)}{u_{cc} w^2 (1 - T')^2 - u_c w^2 T'' + 2u_{ch} w(1 - T') + u_{hh}} \\ &= \underbrace{\frac{u_c w}{d^2 u}}_{\text{Substitution Effect}} \tau'(z^*) + \underbrace{\frac{u_{cc} w(1 - T') + u_{ch}}{d^2 u}}_{\text{Income Effect}} \tau(z^*) \end{aligned} \tag{24}$$

where $d^2 u \equiv u_{cc} w^2 (1 - T')^2 - u_c w^2 T'' + 2u_{ch} w(1 - T') + u_{hh}$ represents the second (total) derivative of utility with respect to h. Equation (24) is essentially a non-linear version of the Slutsky equation and decomposes the behavioral impact of an arbitrary, non-linear tax change into the substitution effect of a change in the marginal tax rate (holding the tax level constant), and the income effect of a change in the tax level (holding the marginal tax rate constant). Dividing both sides by h^*, we can rewrite Equation (24) as:

$$\frac{\partial \log(h^*)}{\partial \mu} = \underbrace{\frac{u_c w}{-d^2 u} \frac{1 - T'(wh^*)}{h^*}}_{\text{Substitution Elasticity, } \xi_{h^*}} \frac{-\tau'(wh^*)}{1 - T'(wh^*)} + \underbrace{\frac{wh^* - T(wh^*)}{h^*} \frac{u_{cc} w(1 - T') + u_{ch}}{-d^2 u}}_{\text{Income Elasticity, } \eta_{h^*}} \frac{-\tau(wh^*)}{wh^* - T(wh^*)} \tag{25}$$

Equation (25) tells us that the behavioral impacts of small tax changes to a non-linear tax schedule can be decomposed as a substitution elasticity ξ_{h^*} multiplied by the percentage change in the keep rate (i.e., one minus the marginal tax rate) at optimal income wh^* as $\frac{-\tau'(wh^*)}{1 - T'(wh^*)} = \frac{\partial \log(1 - T'(wh^*) - \mu\tau'(wh^*))}{\partial \mu}|_{\mu=0}$, and an income elasticity parameter η_{h^*} multiplied by the percentage change in after tax income at optimal income wh^* as $\frac{-\tau(z^*)}{z^* - T(z^*)} = \frac{\partial \log(wh^* - T(wh^*) - \mu\tau(wh^*))}{\partial \mu}|_{\mu=0}$. Because ξ_{h^*} is the elasticity of frictionless hours worked with respect to the marginal tax rate holding the tax level constant, this is a Hicksian elasticity.[24]

Finally, using $d^2 u \equiv u_{cc} w^2 (1 - T')^2 - u_c w^2 T'' + 2u_{ch} w(1 - T') + u_{hh}$, the definition of ξ_{h^*}

[24] Changing marginal tax rates while holding the level of taxes paid constant is equivalent to changing marginal tax rates while holding utility constant by the envelope theorem.

from Equation (25), and Equation (5) yields the statement of the Lemma.

A.3 Proof of Proposition 2

Proof. For individuals for whom $h^* > h^F$, we have that:

$$u(c(h^*), h^F) - u(c(h^*), h^*) > 0$$

given that labor is presumed costly. Given that by definition:

$$u(c(h^F) + \delta(h^F), h^F) - u(c(h^*), h^*) = 0$$

and

$$u(c(h^F), h^F) - u(c(h^*), h^*) < 0$$

then the intermediate value theorem implies that $\delta(h^F) \in (0, c(h^*) - c(h^F)) = (0, wh^* - T(wh^*) - [wh^F - T(wh^F)])$.

For individuals with $h^* < h^F$ we have that:

$$u(c(h^*), 0) - u(c(h^*), h^*) > 0$$

given that labor is presumed costly. By definition:

$$u(c(0) + \delta(0), 0) - u(c(h^*), h^*) = 0$$

and because $u(c(0), 0) < u(c(h^F), h^F)$ by revealed preference as individuals are assumed to have the choice to not work we have that:

$$u(c(0), 0) - u(c(h^*), h^*) < u(c(h^F), h^F) - u(c(h^*), h^*) < 0$$

then the intermediate value theorem implies that $\delta(0) \in (0, c(h^*) - c(0)) = (0, wh^* - T(wh^*) - [0 - T(0)])$. By assumption $\frac{d\delta}{dh}(h) < 0 \; \forall \; h < h^*$ so that $\delta(0) > \delta(h^F)$ as $h > 0$. □

A.4 Proof of Proposition 4

We show that Proposition 1 can be generalized to settings when individuals make multiple choices of hours worked (e.g., spousal labor supply) and consumption choices. Suppose that individuals have m hours choices $h_1, h_2, ..., h_m$ each with wage $w_1, w_2, ..., w_m$. We assume that there is a linear tax rate T_i' that applies to earnings from source i and that individuals have some unearned income R.[25] Suppose individuals have a smooth utility function. The budget constraint for the individual is given by:

$$c = [(1 - T_1')w_1 h_1 + (1 - T_2')w_2 h_2 + ... + (1 - T_m')w_m h_m] + R$$

To condense notation a bit, less us denote the after-tax price of hours as: $p_i \equiv w_i(1 - T_i')$. Defining c as a function of $(h_1, h_2 ..., h_m) \equiv \mathbf{h}$ via $c(\mathbf{h}) = [p_1 h_1 + p_2 h_2 + ... + p_m h_m] + R$ we are

[25] We assume that tax rates are constant solely to make the subsequent matrix expressions simpler but we can allow for a non-linear tax schedule exactly as in Proposition 1.

interested in the cost of frictions $\delta(\mathbf{h}^F)$ defined as:

$$u(c(\mathbf{h}^F)+\delta(\mathbf{h}^F),h_1^F,...,h_m^F)=u(c^*(\mathbf{h}^*),h_1^*,...,h_m^*)$$

where $\mathbf{h}^F=(h_1^F,h_2^F,...,h_m^F)$ denotes hours chosen under frictions. Identical fundamental theorem of calculus arguments as in the proof to Proposition 1 can be used to show that a second order approximation for $\delta(\mathbf{h}^F)$ is given by:[26]

$$\delta(\mathbf{h}^F)\approx\frac{1}{2}(\mathbf{h}^F-\mathbf{h}^*)^T\frac{-\mathbf{H}(\mathbf{h}^*)}{u_c(\mathbf{h}^*)}(\mathbf{h}^F-\mathbf{h}^*) \tag{26}$$

where $\mathbf{H}(\mathbf{h}^*)$ is the Hessian matrix of $u(c(\mathbf{h}),h_1,...,h_m)$ with respect to $\mathbf{h}=(h_1,h_2,...,h_m)$ evaluated at $\mathbf{h}^*=(h_1^*,h_2^*,...,h_m^*)$. Next, we establish that we can recover $\frac{-\mathbf{H}(\mathbf{h}^*)}{u_c(\mathbf{h}^*)}$ from the matrix of (observable) compensated elasticities.

Proof. The system of individual first order conditions expressed in matrix notation is (recognizing that $c=[p_1h_1+p_2h_2+...+p_mh_m]+R$):

$$\begin{aligned}
U_{h_1}&\equiv u_c([p_1h_1^*+...+p_mh_m^*]+R,h_1^*,...,h_m^*)p_2+u_{h_1}([p_1h_1^*+...+p_mh_m^*]+R,h_1^*,...,h_m^*)=0\\
U_{h_2}&\equiv u_c([p_1h_1^*+...+p_mh_m^*]+R,h_1^*,...,h_m^*)p_3+u_{h_2}([p_1h_1^*+...+p_mh_m^*]+R,h_1^*,...,h_m^*)=0\\
&\vdots\\
U_{h_m}&\equiv u_c([p_1h_1^*+...+p_mh_m^*]+R,h_1^*,...,h_m^*)p_m+u_{h_m}([p_1h_1^*+...+p_mh_m^*]+R,h_1^*,...,h_m^*)=0
\end{aligned} \tag{27}$$

Let us apply the implicit function theorem to System (27) to compute the (uncompensated) Slutsky matrix:

$$\begin{bmatrix} U_{h_1h_1} & U_{h_1h_2} & \cdots \\ \vdots & \ddots & \vdots \\ U_{h_mh_1} & \cdots & U_{h_mh_m}\end{bmatrix}\begin{bmatrix} \frac{\partial h_1^*}{\partial p_1} & \frac{\partial h_1^*}{\partial p_2} & \cdots \\ \vdots & \ddots & \vdots \\ \frac{\partial h_m^*}{\partial p_1} & \cdots & \frac{\partial h_m^*}{\partial p_m}\end{bmatrix}=-\begin{bmatrix} U_{h_1p_1} & U_{h_1p_2} & \cdots \\ \vdots & \ddots & \vdots \\ U_{h_mp_1} & \cdots & U_{h_mp_m}\end{bmatrix} \tag{28}$$

where $U_{h_ip_j}$ corresponds to the partial derivative of U_{h_i} with respect to p_j. Note that $\mathbf{H}(\mathbf{h}^*)$ is given by:

$$\mathbf{H}(\mathbf{h}^*)=\begin{bmatrix} U_{h_1h_1} & U_{h_1h_2} & \cdots \\ \vdots & \ddots & \vdots \\ U_{h_mh_1} & \cdots & U_{h_mh_m}\end{bmatrix}$$

To compute the income effect parameter vector, we also apply the implicit function theorem, where U_{h_iR} is the partial derivative of U_{h_i} with respect to unearned income R:

$$\begin{bmatrix} U_{h_1h_1} & U_{h_1h_2} & \cdots \\ \vdots & \ddots & \vdots \\ U_{h_mh_1} & \cdots & U_{h_mh_m}\end{bmatrix}\begin{bmatrix} \frac{\partial h_1^*}{\partial R} \\ \vdots \\ \frac{\partial h_m^*}{\partial R}\end{bmatrix}=-\begin{bmatrix} U_{h_1R} \\ \vdots \\ U_{h_mR}\end{bmatrix} \tag{29}$$

[26]Essentially, parametrize the line segment between h^F and h^*, apply Proposition 1 for this single parameter and then apply the multivariable chain rule; the proof is nearly identical to the proof of the the multivariable version of Taylor's Theorem.

Next, we note that by direct computation one has the relationships:

$$U_{h_i p_i} = u_c + U_{h_i R} h_i^*$$
$$U_{h_i p_j} = U_{h_i R} h_j^*$$
$$U_{h_i R} = u_{cc} p_i + u_{ch_i}$$

Thus, we get the usual Slutsky equations (in vector form) that state, for example:

$$\frac{1}{h_1^*}\begin{bmatrix} \frac{\partial h_1^*}{\partial p_1}|_C \\ \frac{\partial h_2^*}{\partial p_1}|_C \\ \vdots \\ \frac{\partial h_m^*}{\partial p_1}|_C \end{bmatrix} \equiv \frac{1}{h_1^*}\begin{bmatrix} \frac{\partial h_1^*}{\partial p_1} \\ \frac{\partial h_2^*}{\partial p_1} \\ \vdots \\ \frac{\partial h_m^*}{\partial p_1} \end{bmatrix} - \begin{bmatrix} \frac{\partial h_1^*}{\partial R} \\ \frac{\partial h_2^*}{\partial R} \\ \vdots \\ \frac{\partial h_m^*}{\partial R} \end{bmatrix} = -\begin{bmatrix} U_{h_1h_1} & U_{h_1h_2} & \dots \\ \vdots & \ddots & \vdots \\ U_{h_mh_1} & \dots & U_{h_mh_m} \end{bmatrix}^{-1} \begin{bmatrix} \frac{U_{h_1p_1}}{h_1} - U_{h_1R} \\ \vdots \\ \frac{U_{h_mp_1}}{h_1} - U_{h_mR} \end{bmatrix}$$
$$= -\begin{bmatrix} U_{h_1h_1} & U_{h_1h_2} & \dots \\ \vdots & \ddots & \vdots \\ U_{h_mh_1} & \dots & U_{h_mh_m} \end{bmatrix}^{-1} \begin{bmatrix} \frac{u_c}{h_1^*} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{30}$$

The left hand side is just the vector of compenstated (Hicksian) price effects. For clarity, the corresponding equation for the compensated price effects with respect to p_2 equals:

$$\frac{1}{h_2^*}\begin{bmatrix} \frac{\partial h_1^*}{\partial p_2}|_C \\ \frac{\partial h_2^*}{\partial p_2}|_C \\ \vdots \\ \frac{\partial h_m^*}{\partial p_2}|_C \end{bmatrix} \equiv \frac{1}{h_2^*}\begin{bmatrix} \frac{\partial h_1^*}{\partial p_2} \\ \frac{\partial h_2^*}{\partial p_2} \\ \vdots \\ \frac{\partial h_m^*}{\partial p_2} \end{bmatrix} - \begin{bmatrix} \frac{\partial h_1^*}{\partial R} \\ \frac{\partial h_2^*}{\partial R} \\ \vdots \\ \frac{\partial h_m^*}{\partial R} \end{bmatrix} = -\begin{bmatrix} U_{h_1h_1} & U_{h_1h_2} & \dots \\ \vdots & \ddots & \vdots \\ U_{h_mh_1} & \dots & U_{h_mh_m} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \frac{u_c}{h_2^*} \\ \vdots \\ 0 \end{bmatrix} \tag{31}$$

Next, consider the following matrix of compensated semi-elasticities (which are easily recoverable from the matrix of elasticities):

$$A \equiv \begin{bmatrix} \frac{1}{h_1^*}\begin{bmatrix} \frac{\partial h_1^*}{\partial p_1}|_C \\ \frac{\partial h_2^*}{\partial p_1}|_C \\ \vdots \\ \frac{\partial h_m^*}{\partial p_1}|_C \end{bmatrix} & \frac{1}{h_2^*}\begin{bmatrix} \frac{\partial h_1^*}{\partial p_2}|_C \\ \frac{\partial h_2^*}{\partial p_2}|_C \\ \vdots \\ \frac{\partial h_m^*}{\partial p_2}|_C \end{bmatrix} & \dots & \frac{1}{h_m^*}\begin{bmatrix} \frac{\partial h_1^*}{\partial p_m}|_C \\ \frac{\partial h_2^*}{\partial p_m}|_C \\ \vdots \\ \frac{\partial h_m^*}{\partial p_m}|_C \end{bmatrix} \end{bmatrix}$$
$$= -\begin{bmatrix} U_{h_1h_1} & U_{h_1h_2} & \dots \\ \vdots & \ddots & \vdots \\ U_{h_mh_1} & \dots & U_{h_mh_m} \end{bmatrix}^{-1} \begin{bmatrix} \frac{u_c}{h_1^*} & 0 & \dots & 0 \\ 0 & \frac{u_c}{h_2^*} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \frac{u_c}{h_m^*} \end{bmatrix} \tag{32}$$

Inverting both sides of the above equation:

$$A^{-1} = -\begin{bmatrix} \frac{h_1^*}{u_c} & 0 & \dots & 0 \\ 0 & \frac{h_2^*}{u_c} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \frac{h_m^*}{u_c} \end{bmatrix} \begin{bmatrix} U_{h_1h_1} & U_{h_1h_2} & \dots \\ \vdots & \ddots & \vdots \\ U_{h_mh_1} & \dots & U_{h_mh_m} \end{bmatrix} = \begin{bmatrix} h_1^* & 0 & \dots & 0 \\ 0 & h_2^* & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & h_m^* \end{bmatrix} \frac{-\mathbf{H}(\mathbf{h}^*)}{u_c(\mathbf{h}^*)} \tag{33}$$

Yielding:

$$
-\frac{\mathbf{H}(\mathbf{h}^*)}{u_c(\mathbf{h}^*)} = \begin{bmatrix} h_1^* & 0 & \dots & 0 \\ 0 & h_2^* & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & h_m^* \end{bmatrix}^{-1} \left[\frac{1}{h_1^*} \begin{bmatrix} \frac{\partial h_1^*}{\partial p_1}|_C \\ \frac{\partial h_2^*}{\partial p_1}|_C \\ \vdots \\ \frac{\partial h_m^*}{\partial p_1}|_C \end{bmatrix} \quad \frac{1}{h_2^*} \begin{bmatrix} \frac{\partial h_1^*}{\partial p_2}|_C \\ \frac{\partial h_2^*}{\partial p_2}|_C \\ \vdots \\ \frac{\partial h_m^*}{\partial p_2}|_C \end{bmatrix} \quad \dots \quad \frac{1}{h_m^*} \begin{bmatrix} \frac{\partial h_1^*}{\partial p_m}|_C \\ \frac{\partial h_2^*}{\partial p_m}|_C \\ \vdots \\ \frac{\partial h_m^*}{\partial p_m}|_C \end{bmatrix} \right]^{-1} \tag{34}
$$

□

Proposition 4 shows that we can recover the utility costs of frictions in substantially more general settings as long as we can observe the matrix of compensated price effects. While this is useful in situations where we can feasibly estimate the Slutsky matrix of all labor supply decisions, the chief limitation of Proposition 4 is that it requires estimation of a large set of statistics which may be difficult or impossible to estimate.

A.5 Proof of Proposition 5

This section will show how we can construct a lower bound for the value of frictions when households have many labor supply decisions using "aggregate" elasticities. Consider the following labor supply model where household makes multiple labor supply decisions $(h_1, ..., h_j) = \mathbf{h}$ which are converted into labor income via a production function $z(\mathbf{h}) = \sum_i w_i h_i$. Individuals are taxed according to a schedule $T(z)$. The household problem is thus:[27]

$$
\begin{aligned} &\max_{c,\mathbf{h}} \ \tilde{u}(c, \mathbf{h}) \\ &\text{s.t. } \ c \leq z(\mathbf{h}) - T(z(\mathbf{h})) \end{aligned} \tag{35}
$$

Towards constructing a lower bound for the cost of frictions, let us consider the following two-stage maximization problem:

$$
\begin{aligned} &\max_{c,z} \ u(c, z) \\ &\text{s.t. } \ c \leq z - T(z) \end{aligned} \tag{36}
$$

with

$$
\begin{aligned} &u(c, z) = \max_{c,\mathbf{h}} \ \tilde{u}(c, \mathbf{h}) \\ &\text{s.t. } \ c \leq z - T(z) \text{ and } z(\mathbf{h}) = z \end{aligned} \tag{37}
$$

Next, let us define the cost of frictions $\delta(z^F)$ that lead households to earn income z^F rather than z^* assuming that hours are chosen optimally via sub-problem (37):

$$
u(c(z^F) + \delta(z^F), z^F) = u(c(z^*), z^*)
$$

[27] Note, if we can observe (and have exogenous variation in) the wages w_i, then we could apply the multidimensional procedure from Section 4.2 using the full set of (compensated) elasticities of all choice variables $\mathbf{h}$ with respect to the full set of wages.

We can immediately apply Proposition 1 (simply replacing h^F by z^F everywhere) to recover an approximation for $\delta(z^F)$ as long as $\frac{d^3\delta}{dz^3} \approx 0$:

$$\frac{\delta(z^F)}{z^*} \approx (1 - T'(z^*)) \frac{\left(\frac{z^F - z^*}{z^*}\right)^2}{2\xi_{z^*}} \tag{38}$$

where ξ_{z^*} is the elasticity of z^* with respect to the marginal tax rate, defined analogously as in Equation (25).

In actuality, individuals earning an income z^F due to frictions may also be constrained so that their hours choices conditional on an earnings level z^F may be inferior to their conditional optimum given by sub-problem (37). Intuitively, households may ideally prefer to earn the same income z^F with a different mix of labor supply. Hence, for any observed values of $\mathbf{h}^F$ satisfying $z(\mathbf{h}^F) = z^F$ (and therefore $c(\mathbf{h}^F) = c(z^F)$) we will show that

$$\delta(\mathbf{h}^F) > \delta(z^F)$$

where

$$\tilde{u}(c(\mathbf{h}^F) + \delta(\mathbf{h}^F), \mathbf{h}^F) = \tilde{u}(c(\mathbf{h}^*), \mathbf{h}^*) = u(c(z^*), z^*)$$

as long as utility is additively separable with $\tilde{u}(c, \mathbf{h}) = \tilde{u}_{(1)}(c) - \tilde{u}_{(2)}(\mathbf{h})$. Under additive separability:

$$\delta(\mathbf{h}^F) = \tilde{u}_{(1)}^{-1}\left[u(c(z^*), z^*) + \tilde{u}_{(2)}(\mathbf{h}^F)\right] - c(\mathbf{h}^F)$$
$$\delta(z^F) = \tilde{u}_{(1)}^{-1}\left[u(c(z^*), z^*) + \tilde{u}_{(2)}(\mathbf{h}(z^F))\right] - c(z^F)$$

where $\mathbf{h}(z^F)$ represents $\mathbf{h}$ chosen according to sub-problem (37). Hence, disutility of labor is greater under $\mathbf{h}^F$ than $\mathbf{h}(z^F)$ so $\tilde{u}_{(2)}(\mathbf{h}(z^F)) < \tilde{u}_{(2)}(\mathbf{h}^F)$. Recognizing that $c(\mathbf{h}^F) = c(z^F)$ and $\tilde{u}_{(1)}^{-1}$ is strictly increasing (as utility of consumption $\tilde{u}_{(1)}$ is strictly increasing) we have that $\delta(\mathbf{h}^F) > \delta(z^F)$. Thus, we have the following lower bound for $\delta(\mathbf{h}^F)$ (which holds with $\frac{d^3\delta}{dz^3} \approx 0$, which in turn is implied by $\frac{d^3\delta}{d\mathbf{h}^3} \approx 0$ because the former is just a third directional derivative in the direction of a particular vector $\mathbf{h}$):

$$\delta(\mathbf{h}^F) > (1 - T'(z^*)) \frac{\left(\frac{z^F - z^*}{z^*}\right)^2}{2\xi_{z^*}} \tag{39}$$

A.6 Proof of Proposition 3

Towards showing how to (approximately) measure $\delta(h^F, e^F)$ with the difference between h^F and h^* and a set of observable elasticities, we will first establish the following Lemma:

Lemma 2. *As long as all of the third derivatives of δ w.r.t. h and e are small then the WTP to remove frictions expressed as a fraction of optimal income $z^* = wh^*e^*$ can be approximated as:*

$$\frac{\delta(h^F, e^F)}{z^*} \approx -\frac{U_{hh}}{z^* u_c} \frac{(h^*)^2}{2} \left(\frac{h^F - h^*}{h^*}\right)^2 - \frac{U_{ee}}{z^* u_c} \frac{(e^*)^2}{2} \left(\frac{e^F - e^*}{e^*}\right)^2 - \frac{U_{eh}}{z^* u_c} e^* h^* \frac{e^F - e^*}{e^*} \frac{h^F - h^*}{h^*} \tag{40}$$

where U_{hh}, U_{he}, and U_{ee} are second derivatives of the utility function at the optimum (h^*, e^*):

$$U_{hh} = u_{cc}[(1-T')we^*]^2 - u_c(we^*)^2T'' + 2u_{ch}(1-T')we^* + u_{hh}$$
$$U_{he} = u_{cc}(1-T')^2w^2e^*h^* - u_cw^2e^*h^*T'' + u_{ce}(1-T')we^* + u_{ch}(1-T')wh^* + u_c(1-T')w + u_{he}$$
$$U_{ee} = u_{cc}[(1-T')wh^*]^2 - u_c(wh^*)^2T'' + 2u_{ce}(1-T')wh^* + u_{ee}$$

Proof. As in Section 2, let us first show how δ changes as h and e deviate from h^* and e^* by applying the implicit function theorem to Equation (13) to calculate:

$$\frac{\partial\delta}{\partial h}\Big|_{h^*,e^*} = -\frac{u_c(c(h^*,e^*),h^*,e^*)we^*(1-T') + u_h(c(h^*,e^*),h^*,e^*)}{u_c(c(h^*,e^*),h^*,e^*)} = 0 \quad (41)$$

$$\frac{\partial\delta}{\partial e}\Big|_{h^*,e^*} = -\frac{u_c(c(h^*,e^*),h^*,e^*)wh^*(1-T') + u_e(c(h^*,e^*),h^*,e^*)}{u_c(c(h^*,e^*),h^*,e^*)} = 0 \quad (42)$$

where both expressions in Equations (41) and (42) equal 0 by the FOC w.r.t. h and e respectively evaluated at h^* and e^*. Or, more compactly:

$$u_c\frac{\partial\delta}{\partial h} + U_h = 0 \quad (43)$$

$$u_c\frac{\partial\delta}{\partial e} + U_e = 0 \quad (44)$$

Differentiating Equations (43) and (44) and using the fact that $\frac{\partial\delta}{\partial h}\big|_{h^*,e^*} = \frac{\partial\delta}{\partial e}\big|_{h^*,e^*} = 0$, we get that:

$$\frac{\partial^2\delta}{\partial h^2}\Big|_{h^*,e^*} = -\frac{U_{hh}}{u_c} \quad (45)$$
$$\frac{\partial^2\delta}{\partial e^2}\Big|_{h^*,e^*} = -\frac{U_{ee}}{u_c} \quad (46)$$
$$\frac{\partial^2\delta}{\partial h\partial e}\Big|_{h^*,e^*} = -\frac{U_{eh}}{u_c} \quad (47)$$

As a result of Equations (41) and (42), we can use the fundamental theorem of calculus twice to deduce:

$$\begin{aligned}\delta(h^F,e^F) &= \int_{h^*}^{h^F}\frac{\partial\delta}{\partial h}(s,e^*)ds + \int_{e^*}^{e^F}\frac{\partial\delta}{\partial e}(h^F,s)ds \\ &= \int_{h^*}^{h^F}\int_{h^*}^{s}\frac{\partial^2\delta}{\partial h^2}(t,e^*)dtds + \int_{e^*}^{e^F}\left(\int_{e^*}^{s}\frac{\partial^2\delta}{\partial e^2}(h^*,t)dt + \int_{h^*}^{h^F}\frac{\partial^2\delta}{\partial h\partial e}(t,s)dt\right)ds\end{aligned} \quad (48)$$

If third derivatives of δ are small then the second derivatives of δ are $\approx$ constant so that:

$$\begin{aligned}\delta(h^F,e^F) &\approx \frac{\partial^2\delta}{\partial h^2}\Big|_{h^*,e^*}\int_{h^*}^{h^F}\int_{h^*}^{s}dtds + \frac{\partial^2\delta}{\partial e^2}\Big|_{h^*,e^*}\int_{e^*}^{e^F}\int_{e^*}^{s}dtds + \frac{\partial^2\delta}{\partial h\partial e}\Big|_{h^*,e^*}\int_{e^*}^{e^F}\int_{h^*}^{h^F}dtds \\ &= -\frac{U_{hh}}{2u_c}\left(h^F-h^*\right)^2 - \frac{U_{ee}}{2u_c}\left(e^F-e^*\right)^2 - \frac{U_{eh}}{u_c}(e^F-e^*)(h^F-h^*)\end{aligned} \quad (49)$$

Dividing Equation (49) by z^* and multiplying the three terms by $\frac{(h^*)^2}{(h^*)^2}$, $\frac{(e^*)^2}{(e^*)^2}$, and $\frac{h^*e^*}{h^*e^*}$ (respectively) yields the statement of the Lemma. □

Next, using Lemma 2, we can prove Proposition 3:

Proof. The first step is to find expressions for the ideal hours elasticity w.r.t. the marginal tax rate, ξ_{h^*}, in terms of second derivatives of the utility function. Let us first consider utility as a function of an arbitrary non-linear tax schedule perturbation $\tau(wh^*e^*)$:

$$u(wh^*e^* - T(wh^*e^*) - \epsilon\tau(wh^*e^*), h^*, e^*)$$

First, at the optimal bundle we know that the following two FOCs are satisfied (assuming the tax schedule is differentiable):

$$\begin{aligned}
&u_c(wh^*e^* - T(wh^*e^*) - \epsilon\tau(wh^*e^*), h^*, e^*)[1 - T'(wh^*e^*) - \epsilon\tau'(wh^*e^*)]we^* \\
&+ u_h(wh^*e^* - T(wh^*e^*) - \epsilon\tau(wh^*e^*), h^*, e^*) = 0 \\
&u_c(wh^*e^* - T(wh^*e^*) - \epsilon\tau(wh^*e^*), h^*, e^*)[1 - T'(wh^*e^*) - \epsilon\tau'(wh^*e^*)]wh^* \\
&+ u_e(wh^*e^* - T(wh^*e^*) - \epsilon\tau(wh^*e^*), h^*, e^*) = 0
\end{aligned} \tag{50}$$

Next, let us apply the implicit function theorem to the FOCs in (50) to calculate how optimal hours and optimal effort vary with an arbitrary tax reform in the direction of $\tau(z)$: $\frac{\partial h^*}{\partial \epsilon}$ and $\frac{\partial e^*}{\partial \epsilon}$. Differentiating both equations w.r.t. ϵ and evaluating at $\epsilon = 0$ We have (omitting function arguments for brevity, noting that everything is evaluated at (h^*, e^*)):

$$\begin{aligned}
&- u_c we^*\tau' - u_{cc}(1 - T')we^*\tau - u_{ch}\tau + U_{hh}\frac{\partial h^*}{\partial \epsilon} + U_{he}\frac{\partial e^*}{\partial \epsilon} = 0 \\
&- u_c wh^*\tau' - u_{cc}(1 - T')wh^*\tau - u_{ce}\tau + U_{eh}\frac{\partial h^*}{\partial \epsilon} + U_{ee}\frac{\partial e^*}{\partial \epsilon} = 0
\end{aligned} \tag{51}$$

In the Proposition we assumed that we can observe the elasticity of h^* with respect to the marginal net of tax rate holding the tax level constant. For an individual with optimal income z^* this corresponds to the tax perturbation $\tau(z)$ that decreases marginal rates at z^* but leaves the tax level unchanged: $\tau(z) = (z^* - z)$ so that $\tau'(z^*) = -1$ and $\tau(z^*) = 0$. So let us consider this perturbation, denoting:

$$\xi_{h^*} \equiv \frac{\partial h^*}{\partial \epsilon}\frac{1 - T'}{h^*}\bigg|_{\tau(z)=(z^*-z)}$$

$$\xi_{e^*} \equiv \frac{\partial e^*}{\partial \epsilon}\frac{1 - T'}{e^*}\bigg|_{\tau(z)=(z^*-z)}$$

Hence, for the perturbation direction $\tau(z) = (z^* - z)$ at $z = z^*$, we have:

$$\begin{aligned}
&u_c we^* + U_{hh}\frac{h^*}{1 - T'}\xi_{h^*} + U_{he}\frac{e^*}{1 - T'}\xi_{e^*} = 0 \\
&u_c wh^* + U_{eh}\frac{h^*}{1 - T'}\xi_{h^*} + U_{ee}\frac{e^*}{1 - T'}\xi_{e^*} = 0
\end{aligned} \tag{52}$$

From here, multiply the first equation in (52) by h^* and the second equation by e^* then divide both equations by z^*u_c to yield:

$$\begin{aligned}
&1 + \frac{U_{hh}}{z^*u_c}\frac{(h^*)^2}{1 - T'}\xi_{h^*} + \frac{U_{he}}{z^*u_c}\frac{h^*e^*}{1 - T'}\xi_{e^*} = 0 \\
&1 + \frac{U_{he}}{z^*u_c}\frac{h^*e^*}{1 - T'}\xi_{h^*} + \frac{U_{ee}}{z^*u_c}\frac{(e^*)^2}{1 - T'}\xi_{e^*} = 0
\end{aligned} \tag{53}$$

Solving Equation (53), we have the following expression for ξ_{h^*}:

$$\xi_{h^*} = \frac{\frac{U_{he}}{z^* u_c}\frac{h^* e^*}{1-T'} - \frac{U_{ee}}{z^* u_c}\frac{(e^*)^2}{1-T'}}{\frac{U_{hh}}{z^* u_c}\frac{(h^*)^2}{1-T'}\frac{U_{ee}}{z^* u_c}\frac{(e^*)^2}{1-T'} - \left[\frac{U_{he}}{z^* u_c}\frac{h^* e^*}{1-T'}\right]^2} \tag{54}$$

Next, we will utilize the assumption that effort is chosen without any frictions. Specifically, we consider how the effort decision changes as hours change exogenously. First, the effort decision e always satisfies the following FOC for any given exogenously determined h because effort is chosen without any frictions:

$$u_c(whe - T(whe), h, e)(1 - T')wh + u_e(whe - T(whe), h, e) = 0$$

Applying the implicit function to the above FOC we have that:

$$\frac{de}{dh} = -\frac{U_{he}}{U_{ee}}$$

or in terms of elasticities:

$$\frac{d\log(e)}{d\log(h)} = \frac{de}{dh}\frac{h}{e} = -\frac{U_{he}h}{U_{ee}e} \tag{55}$$

Thus, let us consider how how observed income changes with an exogenous shift in actual hours h as we move away from h^*, holding w and $T(\cdot)$ (and therefore ideal hours h^*) constant:

$$\chi_{z,h} \equiv \left.\frac{d\log(z)}{d\log(h)}\right|_{h^*} = \left.\frac{d\log(we(h)h)}{d\log(h)}\right|_{h^*} = 1 + \left.\frac{d\log(e)}{d\log(h)}\right|_{h^*} = 1 - \frac{U_{he}/(z^* u_c)h^* e^*}{U_{ee}/(z^* u_c)(e^*)^2} \tag{56}$$

Next, we can take of a Taylor series of $\log(e(h))$ around $\log(h^*)$ using Equation (55):

$$\log(e) = \log(e^*) - \frac{U_{he}/(z^* u_c)h^* e^*}{U_{ee}/(z^* u_c)(e^*)^2}\left[\log(h) - \log(h^*)\right] \tag{57}$$

We can use Equation (57) to infer (a first order approximation for) $\frac{e^F - e^*}{e^*}$ from $\frac{h^F - h^*}{h^*}$:

$$\begin{aligned}\frac{e^F - e^*}{e^*} &\approx \log(e(h^F)) - \log(e^*) = -\frac{U_{he}/(z^* u_c)h^* e^*}{U_{ee}/(z^* u_c)(e^*)^2}\left[\log(h^F) - \log(h^*)\right] \\ &\approx -\frac{U_{he}/(z^* u_c)h^* e^*}{U_{ee}/(z^* u_c)(e^*)^2}\frac{h^F - h^*}{h^*} = (\chi_{z,h} - 1)\frac{h^F - h^*}{h^*}\end{aligned} \tag{58}$$

Next, let us rearrange our expression for $\frac{\delta(h^F, e^F)}{z^*}$ from Equation (40) slightly:

$$\begin{aligned}&-\frac{U_{hh}(h^*)^2}{2z^* u_c}\left(\frac{h^F - h^*}{h^*}\right)^2 - \frac{U_{ee}(e^*)^2}{2z^* u_c}\left(\frac{e^F - e^*}{e^*}\right)^2 - \frac{U_{eh}h^* e^*}{z^* u_c}\frac{e^F - e^*}{e^*}\frac{h^F - h^*}{h^*} \\ &= \left\{-\frac{U_{hh}(h^*)^2}{2z^* u_c} - \frac{U_{ee}(e^*)^2}{2z^* u_c}(\chi_{z,h} - 1)^2 - \frac{U_{eh}h^* e^*}{z^* u_c}(\chi_{z,h} - 1)\right\}\left(\frac{h^F - h^*}{h^*}\right)^2 \\ &= \left\{-\frac{U_{hh}(h^*)^2}{2z^* u_c} + \frac{U_{eh}h^* e^*}{2z^* u_c}(\chi_{z,h} - 1) - \frac{U_{eh}h^* e^*}{z^* u_c}(\chi_{z,h} - 1)\right\}\left(\frac{h^F - h^*}{h^*}\right)^2 \\ &= \left\{-\frac{U_{hh}(h^*)^2}{2z^* u_c} - \frac{U_{eh}h^* e^*}{2z^* u_c}(\chi_{z,h} - 1)\right\}\left(\frac{h^F - h^*}{h^*}\right)^2\end{aligned} \tag{59}$$

where the first equality substitutes in the Equation for $\frac{e^F - e^*}{e^*}$ from Equation (58), the second equality uses the fact that $(\chi_{z,h} - 1) = -\frac{U_{he}/(z^* u_c)h^* e^*}{U_{ee}/(z^* u_c)(e^*)^2}$, and the third equality just adds terms together.

The final step is to use a bit of algebra to rearrange our expression for the ideal hours elasticity from Equation (54):

$$
\begin{aligned}
(1-T')\frac{1}{\xi_{h^*}} &= \frac{\frac{U_{hh}(h^*)^2}{z^*u_c}\frac{U_{ee}(e^*)^2}{z^*u_c} - \left[\frac{U_{he}h^*e^*}{z^*u_c}\right]^2}{\frac{U_{he}h^*e^*}{z^*u_c} - \frac{U_{ee}(e^*)^2}{z^*u_c}} \\
&= \frac{-\frac{U_{hh}(h^*)^2}{z^*u_c} - \frac{U_{he}h^*e^*}{z^*u_c}\left[-\frac{U_{he}h^*e^*}{z^*u_c}\Big/\frac{U_{ee}(e^*)^2}{z^*u_c}\right]}{\left[-\frac{U_{he}h^*e^*}{z^*u_c}\Big/\frac{U_{ee}(e^*)^2}{z^*u_c}\right]+1} \\
&= \frac{-\frac{U_{hh}(h^*)^2}{z^*u_c} - \frac{U_{he}h^*e^*}{z^*u_c}(\chi_{z,h}-1)}{\chi_{z,h}}
\end{aligned}
\tag{60}
$$

Combining Equations (59) and (60), we get that:

$$
\frac{\delta(h^F,e^F)}{z^*} \approx \left\{-\frac{U_{hh}(h^*)^2}{2z^*u_c} - \frac{U_{eh}h^*e^*}{2z^*u_c}(\chi_{z,h}-1)\right\}\left(\frac{h^F-h^*}{h^*}\right)^2 = (1-T')\frac{\chi_{z,h}}{2\xi_{h^*}}\left(\frac{h^F-h^*}{h^*}\right)^2 \tag{61}
$$

□

A.7 Proof of Proposition 6

Proof. First, let us rewrite the maximization problem (15) with a perturbed tax schedule in period t of $T_t(w_th_t)+\epsilon\tau_t(w_th_t)$:

$$
\max_{h_t}\left\{\max_{\sigma_t} u(w_th_t - T_t(w_th_t) - \epsilon\tau_t(w_th_t) - \sigma_t, h_t) + \mathbb{E}[V(w_{t+1}(h_t), \kappa_t + r_t\sigma_t)]\right\} \tag{62}
$$

Next, let us solve the inner problem. Suppose for the time being that $\sigma_t^*(h_t)$ is interior and let u_1 and V_2 represent derivatives of u and V w.r.t. the first and second arguments:

$$
-u_1(w_th_t - T_t(w_th_t) - \epsilon\tau_t(w_th_t) - \sigma_t^*(h_t), h_t) + \mathbb{E}[V_2(w_{t+1}(h_t), \kappa_t + r_t\sigma_t^*(h_t))] = 0 \tag{63}
$$

Hence, we can rewrite Equation (62) as follows:

$$
\max_{h_t} u(w_th_t - T_t(w_th_t) - \epsilon\tau_t(w_th_t) - \sigma_t^*(h_t), h_t) + \mathbb{E}[V(w_{t+1}(h_t), \kappa_t + r_t\sigma_t^*(h_t)] \tag{64}
$$

Next, we have the first order condition that is satisfied when $h_t = h_t^*$:

$$
\begin{aligned}
&u_1(w_th_t - T_t(w_th_t) - \epsilon\tau_t(w_th_t) - \sigma_t^*(h_t), h_t)w_t(1 - T_t' - \epsilon\tau_t') \\
&+ u_2(w_th_t - T_t(w_th_t) - \epsilon\tau_t(w_th_t) - \sigma_t^*(h_t), h_t) + \mathbb{E}\left[V_1(w_{t+1}(h_t), \kappa_t + r_t\sigma_t^*(h_t))\frac{dw_{t+1}}{dh_t}\right] \\
&+ \left[-u_1(w_th_t - T_t(w_th_t) - \epsilon\tau_t(w_th_t) - \sigma_t^*(h_t), h_t) + \mathbb{E}\left[V_2(w_{t+1}(h_t), \kappa_t + r_t\sigma_t^*(h_t))\right]\right]\frac{\partial\sigma_t^*}{\partial h_t} = 0
\end{aligned}
\tag{65}
$$

Next, let us consider how h_t changes when we perturb the tax schedule in the direction of the function $\tau_t(w_th_t) = (w_t^*h_t^* - w_th_t)$ which decreases marginal tax rates at $w_t^*h_t^*$ but does not change the tax level at $w_t^*h_t^*$ (i.e., a compensated tax change). Next, we apply the implicit function theorem. Differentiating Equation (65) w.r.t. ϵ with $\tau_t(w_th_t) = (w_t^*h_t^* - w_th_t)$ and evaluating at $\epsilon = 0$ and $h_t = h_t^*$, noting that $\tau_t'(w_t^* h_t^*) = -1$, we have that:

$$u_1(w_t h_t^* - T_t(w_t h_t^*) - \sigma_t^*, h_t^*) w_t + U_{h_t h_t}(h_t^*) \frac{\partial h^*}{\partial \epsilon} = 0 \tag{66}$$

where $U_{h_t h_t}(h_t^*)$ represents the second derivative $u(w_t h_t - T_t(w_t h_t) - \epsilon \tau_t(w_t h_t) - \sigma_t^*(h_t), h_t) + \mathbb{E}[V(w_{t+1}(h_t), \kappa_t + r_t \sigma_t^*)]$ w.r.t h_t evaluated at h_t^*. Note, we have used the fact that by the FOC for σ_t:

$$[-u_1(w_t h_t - T_t(w_t h_t) - \epsilon \tau_t(w_t h_t) - \sigma_t^*(h_t), h_t) + \mathbb{E}\left[V_2(w_{t+1}(h_t), \kappa_t + r_t \sigma_t^*(h_t))\right]] = 0$$

so that we can ignore calculating $\frac{\partial}{\partial \epsilon}\left(\frac{\partial \sigma_t^*}{\partial h_t}\right)$ when differentating Equation (65) w.r.t. ϵ. From Equation (66) we have that:

$$\frac{-U_{h_t h_t}(h_t^*)}{u_1(h_t^*)} = w_t \left(\frac{\partial h_t^*}{\partial \epsilon}\right)^{-1} \equiv w_t \left(\xi_{h_t^*} \frac{h_t^*}{1 - T'}\right)^{-1} = \frac{w_t(1 - T')}{\xi_{h_t^*} h_t^*} \tag{67}$$

But what if $\sigma_t^*(h_t)$ is not interior? In this case, we have that $\frac{\partial \sigma_t^*}{\partial h_t} = 0$ as we assume either that $\sigma_t^*(h_t)$ is interior or σ_t^* does not change with small changes in h_t so that Equation (67) still holds.

From here, it should be clear that because we define $\delta(h_t^F)$ as:

$$\begin{aligned} &u(w_t h_t^F - T_t(w_t h_t^F) - \sigma_t^*(h_t^F) + \delta(h_t^F), h_t^F) + \mathbb{E}[V(w_{t+1}(h_t^F), \kappa_t + r_t \sigma_t^*(h_t^F))] \\ &= u(w_t h_t^* - T_t(w_t h_t^*) - \sigma_t^*(h_t^*), h_t^*) + \mathbb{E}[V(w_{t+1}(h_t^*), \kappa_t + r_t \sigma_t^*(h_t^*))] \end{aligned}$$

then the analogue of Equation (5) is that (this follows exactly by the logic of Appendix A.1):

$$\left.\frac{d^2 \delta}{dh^2}\right|_{h_t^*} = \frac{-U_{h_t h_t}(h_t^*)}{u_1(h_t^*)} \tag{68}$$

Putting Equations (67) and (68) together and applying the fundamental theorem of calculus exactly as in Proposition 1 we get Proposition 6. □

B Appendix: Additional Results

B.1 Cost of Friction Bounds with Effort Decision

We can also bound the cost frictions in a model with effort decisions if we add in an additional assumption, letting $z^F \equiv w h^F e^F$:

Proposition 9. *Suppose that there are no frictions in choosing effort per hour and that $e(h)$ represents optimal effort for any given hours. Also suppose individuals always have the choice to not work. The willingness-to-pay to remove frictions is bounded by:*

$$\frac{\delta(h^F, e(h^F))}{z^*} < \frac{z^* - T(z^*) - [z^F - T(z^F)]}{z^*} \qquad \text{for } h^* > h^F \tag{69}$$

$$\frac{\delta(h^F, e(h^F))}{z^*} < \frac{z^* - T(z^*) + T(0)}{z^*} \qquad \text{for } h^* < h^F \tag{70}$$

where $z^* \approx z^F \frac{h^*}{h^F} \frac{1}{1 + (\chi_{z,h} - 1)\frac{h^F - h^*}{h^*}}$ as long as:

- $\frac{du(c,h,e(h))}{dh} < 0 \; \forall c, h$ *so that working more hours is costly, even accounting for changes in effort that occur concurrently (i.e., hours and effort per hour aren't strong substitutes).*
- $\frac{d\delta(h,e(h))}{dh}|_h < 0 \; \forall \; h < h^*$ *so that WTP to remove frictions is increasing as h gets farther away from* h^**.*

Proof. For individuals for whom $h^* > h^F$, we have that:

$$u(c(h^*, e^*), h^F, e(h^F)) - u(c(h^*, e^*), h^*, e^*) > 0$$

given that our first assumption ensures working more hours is costly (and $e^* = e(h^*)$ because effort is chosen frictionlessly). Given that by definition:

$$u(c(h^F, e(h^F)) + \delta(h^F, e(h^F)), h^F, e(h^F)) - u(c(h^*, e^*), h^*, e^*) = 0$$

and

$$u(c(h^F, e(h^F)), h^F, e(h^F)) - u(c(h^*, e^*), h^*, e^*) < 0$$

then the intermediate value theorem implies that $\delta(h^F, e(h^F)) \in (0, c(h^*, e^*) - c(h^F, e(h^F))) = (0, z^* - T(z^*) - [z^F - T(z^F)])$ where $z^* = wh^*e^*$ and $z^F = wh^F e(h^F)$.

For individuals with $h^* < h^F$ we have that:

$$u(c(h^*, e^*), 0, 0) - u(c(h^*, e^*), h^*, e^*) > 0$$

given that h and e are presumed costly. Given that by definition:

$$u(c(0, 0) + \delta(0, 0), 0, 0) - u(c(h^*, e^*), h^*, e^*) = 0$$

and $u(c(0, 0), 0, 0) < u(c(h^F, e(h^F)), h^F, e(h^F))$ by revealed preference as individuals are assumed to have the choice to not work we have that:

$$u(c(0, 0), 0, 0) - u(c(h^*, e^*), h^*, e^*) < u(c(h^F, e(h^F)), h^F, e(h^F)) - u(c(h^*, e^*), h^*, e^*) < 0$$

then the intermediate value theorem implies that $\delta(0, 0) \in (0, c(h^*, e^*) - c(0, 0)) = (0, z^* - T(z^*) - [0 - T(0)])$. By assumption $\frac{d\delta(h,e(h))}{dh}|_h < 0 \; \forall \; h < h^*$ so that $\delta(0, 0) > \delta(h^F, e(h^F))$ as $h^F > 0$ and $e(0) = 0$.

Finally $z^* \approx z^F \frac{h^*}{h^F} \frac{1}{1+(\chi_{z,h}-1)\frac{h^F-h^*}{h^*}}$ follows from the fact that $z^F = wh^F e^F$ and $z^* = wh^*e^* = z^F \frac{h^*}{h^F}\frac{e^*}{e^F}$ and the approximation for $\frac{e^*}{e^F}$ from Equation (58) derived in the proof to Proposition 3. □

The intuition is essentially the same as Proposition 2: for individuals who would ideally like to work more hours than they currently do, the WTP to remove frictions cannot be larger than the increase in your after tax income because labor supply is costly; for individuals who would ideally like to work fewer hours than they currently do, the WTP to remove frictions is bounded above by the WTP of moving from unemployment to her optimum, which is bounded by the consumption gain that she gets from working at her optimal hours relative to unemployment.

B.2 German Tax Reforms

We will explore the impacts of tax changes on the ideal hours worked in the context of a large German tax reform in 1996 that changed marginal tax rates at different rates for most taxpayers. Figure 8 shows the German marginal tax rate schedule in three different time periods. Before 1996, incomes below 5,626 Deutschemark (DM) were taxed at 0%, with a discontinuous jump to 19% for incomes up to 8,153 DM.[28] The marginal tax rate increased linearly with income up to 120,041 DM before again leveling out at a constant 53% marginal tax rate. In 1996, Germany underwent its only significant reform to the marginal rate schedule of the decade, nearly doubling the income threshold for 0% marginal taxes to 12,095 DM and increasing the marginal rate of the first non-zero bracket from 19% to ≈ 26%.[29]

Germany underwent another tax reform in 2001, illustrated by the green line in 8. Prior to 2001, the tax rate increased linearly from 48.5% to 53% between incomes of 107,567 DM 120,041 DM; in 2001 the tax schedule was changed so that the top marginal tax rate remained constant at 48.5% starting at an income of 107,567 DM. There were also some small adjustments at the bottom of the income distribution as well, which are shown in the figure. When estimating the elasticity of h^* with respect to the marginal tax rate, we will leverage all of the variation in tax rates from 1995-2001.

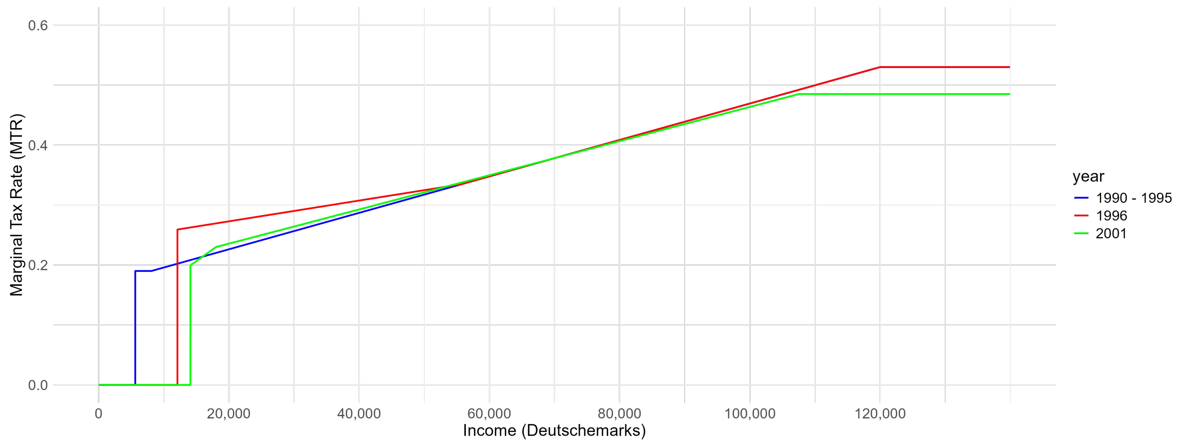


Figure 8: Marginal Tax Rate Schedule in Germany, 1991-2001

Notes: This figure displays the variation in German marginal tax rates over time. The tax schedule was the same from 1991 to 1995, then was reformed in 1996. The tax schedule then underwent small reforms each year between 1998, 1999, and 2000. There was another reform in 2001 which was larger than the reforms in 1998, 1999, and 2000 but smaller than the reform in 1996.

[28] Germany used the Deutschemark before 2002. The conversion factor in 2002 was 1.96 DM/Euro.

[29] There were very small adjustments to the linearly-increasing portion of the schedule in 1998 and 1999, and an additional (small) linearly-increasing tax bracket was also created in 1999.

B.3 Deriving Equation (10)

We begin with Equation (25), which describes behavioral responses to arbitrary non-linear tax perturbations, reproduced below:

$$\frac{\partial \log(h^*)}{\partial \mu} = \underbrace{\frac{u_c w}{-d^2 u} \frac{1 - T'(wh^*)}{h^*}}_{\text{Substitution Elasticity, } \xi_{h^*}} \frac{-\tau'(wh^*)}{1 - T'(wh^*)} + \underbrace{\frac{wh^* - T(wh^*)}{h^*} \frac{u_{cc} w(1 - T') + u_{ch}}{-d^2 u}}_{\text{Income Elasticity, } \eta_{h^*}} \frac{-\tau(wh^*)}{wh^* - T(wh^*)}$$

Recall that for purposes of determining the willingness-to-pay to remove frictions, the relevant parameter is ϵ which captures how h^* changes with the marginal tax rate, holding the tax level constant. Equation (25) implies that the impact of a small tax change to a non-linear tax schedule can be decomposed into a substitution elasticity ξ_{h^*} multiplied by the percentage change in the keep rate at optimal income wh^*, $\frac{-\tau'(wh^*)}{1-T'(wh^*)} = \frac{\partial \log(1-T'(wh^*)-\mu\tau'(wh^*))}{\partial \mu}|_{\mu=0}$, and an income elasticity parameter η_{h^*} multiplied by the percentage change in after tax income at the optimal income wh^*, $\frac{-\tau(z^*)}{z^*-T(z^*)} = \frac{\partial \log(wh^*-T(wh^*)-\mu\tau(wh^*))}{\partial \mu}|_{\mu=0}$. Hence, we have:

$$\frac{\partial \log(h^*)}{\partial \mu} = \xi_{h^*} \frac{\partial \log(1 - T'(wh^*) - \mu\tau'(wh^*))}{\partial \mu} + \eta_{h^*} \frac{\partial \log(wh^* - T(wh^*) - \mu\tau(wh^*))}{\partial \mu} \tag{71}$$

Approximating the derivatives in Equation (71) yields Equation (10). Importantly, note that the terms multiplying the substitution elasticity and the income elasticity are $\frac{\partial \log(1-T'(wh^*)-\mu\tau'(wh^*))}{\partial \mu}$ and $\frac{\partial \log(wh^*-T(wh^*)-\mu\tau(wh^*))}{\partial \mu}$ which capture the *mechanical* changes to the keep rate and after-tax income that result from the tax perturbation, *holding decisions fixed.*

B.4 Inferring Perceived Marginal Tax Rates from Rees-Jones and Taubinsky (2020)

The survey data from Rees-Jones and Taubinsky (2020) asks individuals about their perceived tax liability but not their perceived marginal tax rates. To infer their perceptions about marginal tax rates, we use the fact that individuals were also asked about their perceptions of the tax liability at 16 different income levels other than their own. Hence, we can get an estimate of the perceived marginal tax rate between two income levels z_1 and z_2 with the approximation: $\hat{T}' = \frac{\hat{T}(z_2)-\hat{T}(z_1)}{z_2-z_1}$.

Letting z_1 represent the individual's income, one may be inclined to use the z_2 which is closest to z_1 to infer the individual's perception of their marginal tax rate. However, this turns out to be problematic in this setting. If individuals make errors, σ_i in their assessment of $\hat{T}(z_i)$, then when $|z_2 - z_1| \approx 0$, then $\hat{T}' = \frac{\hat{T}(z_2)-\hat{T}(z_1)+\sigma_2-\sigma_1}{z_2-z_1}$ can be significantly biased if $\sigma_i \perp z_i$ (as the denominator gets very small but the errors do not become correspondingly small). In particular, if individuals are unsure about the tax level at z_1, z_2 they may just guess $T(z_1) = T(z_2)$ if $z_1 \approx z_2$ heuristically; however, this is probably not a good indication of their perception of marginal tax rates and is rather a reflection of their uncertainty about tax liabilities more generally.

Alternatively, individuals may "hedge" their answers by answering that $T(z_2) = T(z_1) + 1000$ if $z_1 \approx z_2$, but this will lead to non-sensical tax rates above 100% if $|z_2 - z_1| < 1000$. Both of these biases seem to be quite common in the survey data of Rees-Jones and Taubinsky (2020), see Figure **??** which plots the distribution of inferred marginal tax rates when using the closest z_2 to each z_1.

Hence, there is a significant concern that using values of z_2 that are very close to z_1 will yield incorrect estimates of individual's perceived marginal tax rates. In contrast, using values of z_2 that are very far from z_1 will conflate their perceptions about marginal tax rates and their perceptions about average tax rates. As a middle ground approach, we use the value of z_2 that is farther away from z_1 yet still in the same tax bracket to infer their perceptions about marginal tax rates; this is what is shown in Figure **??** in the text.

B.5 Decomposition: Adjustment Frictions vs. Misperceptions

From Equation (18), we can decompose the cost of frictions as follows by using the fact that $h^F - h^{**} = (h^F - h^*) + (h^* - h^{**})$:

$$\begin{aligned}\frac{\delta(h^F)}{wh^{**}} &\approx (1-T')\frac{\left(\frac{h^F-h^{**}}{h^{**}}\right)^2}{2\xi_{h^{**}}} \\ &= \left[\underbrace{\frac{1-T'}{2\xi_{h^{**}}}\left(\frac{h^F-h^*}{h^{**}}\right)^2}_{\text{Adjustment Frictions}} + \underbrace{\frac{1-T'}{2\xi_{h^{**}}}\left(\frac{h^*-h^{**}}{h^{**}}\right)^2}_{\text{Misperceptions}} + \underbrace{2\frac{1-T'}{2\xi_{h^{**}}}\frac{h^F-h^*}{h^{**}}\frac{h^*-h^{**}}{h^{**}}}_{\text{Covariance}}\right]\end{aligned} \tag{72}$$

If our estimate of $\frac{\delta(h^F)}{wh^{**}}$ comes from Proposition 2, then we can decompose the impact of adjustment frictions vs. misperceptions as:

$$\begin{aligned}\frac{\delta(h^F)}{wh^{**}} &= \frac{wh^{**} - T(wh^{**}) - [wh^F - T(wh^F)]}{wh^{**}} \\ &= \left[\underbrace{\frac{wh^* - T(wh^*) - [wh^F - T(wh^F)]}{wh^{**}}}_{\text{Adjustment Frictions}} + \underbrace{\frac{wh^{**} - T(wh^{**}) - [wh^* - T(wh^*)]}{wh^{**}}}_{\text{Misperceptions}}\right]\end{aligned} \tag{73}$$

$$\begin{aligned}\frac{\delta(h^F)}{wh^{**}} &= \frac{wh^{**} - T(wh^{**}) + T(0)}{wh^{**}} \\ &= \left[\underbrace{\frac{wh^* - T(wh^*) + T(0)}{wh^{**}}}_{\text{Adjustment Frictions}} + \underbrace{\frac{wh^{**} - T(wh^{**}) - (wh^* - T(wh^*))}{wh^{**}}}_{\text{Misperceptions}}\right]\end{aligned} \tag{74}$$

Using Equations (72), (73), and (74) we can decompose the cumulative cost of frictions into the contributions of the adjustment frictions, misperceptions, and their covariance. Figure 9 presents the results from this exercise setting $\eta = 0$. Overall, we see that at small values of $\xi_{h^{**}}$, misperceptions are essentially not contributing to the cumulative cost of frictions. Even at $\xi_{h^{**}} =$

1, adjustment frictions generate 80% of the cumulative cost of frictions while misperceptions generate the remaining 20%. The covariance has essentially no contribution to the cumulative cost of frictions because the correlation between $\frac{h^F - h^*}{h^{**}}$ and $\frac{h^* - h^{**}}{h^{**}}$ turns out to only be 0.01.[30]

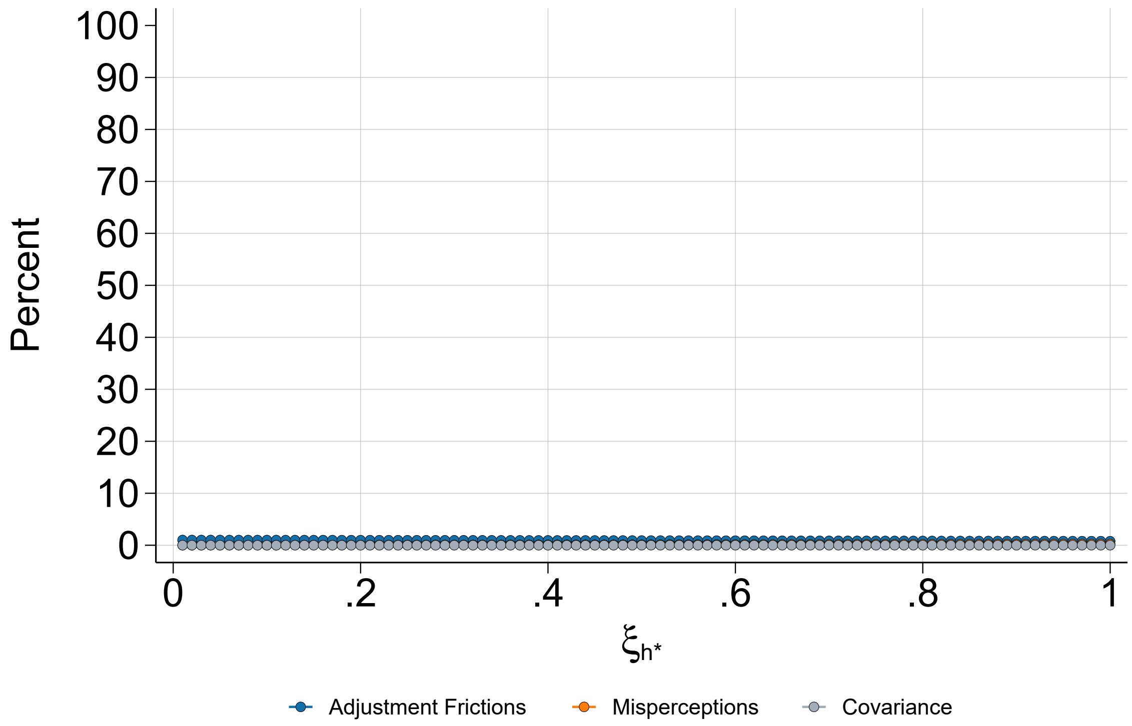


Figure 9: Decomposition of Cost of Frictions

Notes: Cost of frictions w/ $\xi_{h^{**}}$.

B.6 Adjustment Frictions vs. Misperceptions in the GSOEP

We do not have data on the size of misperceptions in Germany. However, we nonetheless explore what the impact of misperceptions might look like in the context of Germany by exploring the possibility that all individuals misperceive their average tax rate as their marginal tax rate (both De Bartolome (1995) and Rees-Jones and Taubinsky (2020) identify that $\approx$ 50% of taxpayers use this heuristic). Figure 10 shows the cumulative size of frictions if all individuals misperceive their average tax rate as their marginal tax rate in the GSOEP (and assuming they correctly perceive their total tax burden). Qualitatively, the takeaways from this exercise are the same as in the NSCW in Section 5.4: (1) adjustment frictions are larger than misperceptions (when $\xi_{h^*} = 0.2$ adjustment frictions account for 89% of the cumulative cost and even when $\xi_{h^*} = 1$ adjustment frictions account for 60% of the cumulative cost) and (2) even though adjustment frictions become smaller with ξ_{h^*}, the cost of misperceptions goes up with ξ_{h^*} so that the

[30]For comparison, if we set $\eta = -0.5$, then misperceptions contribute about 10% to the cumulative cost when $\xi_{h^{**}} = 0.01$ (and the covariance contributes about -7%) and misperceptions contribute about 19% when $\xi_{h^{**}} = 1$ (and the covariance contributes about -1%).

cumulative cost of frictions is large no matter the value of ξ_{h^*}.

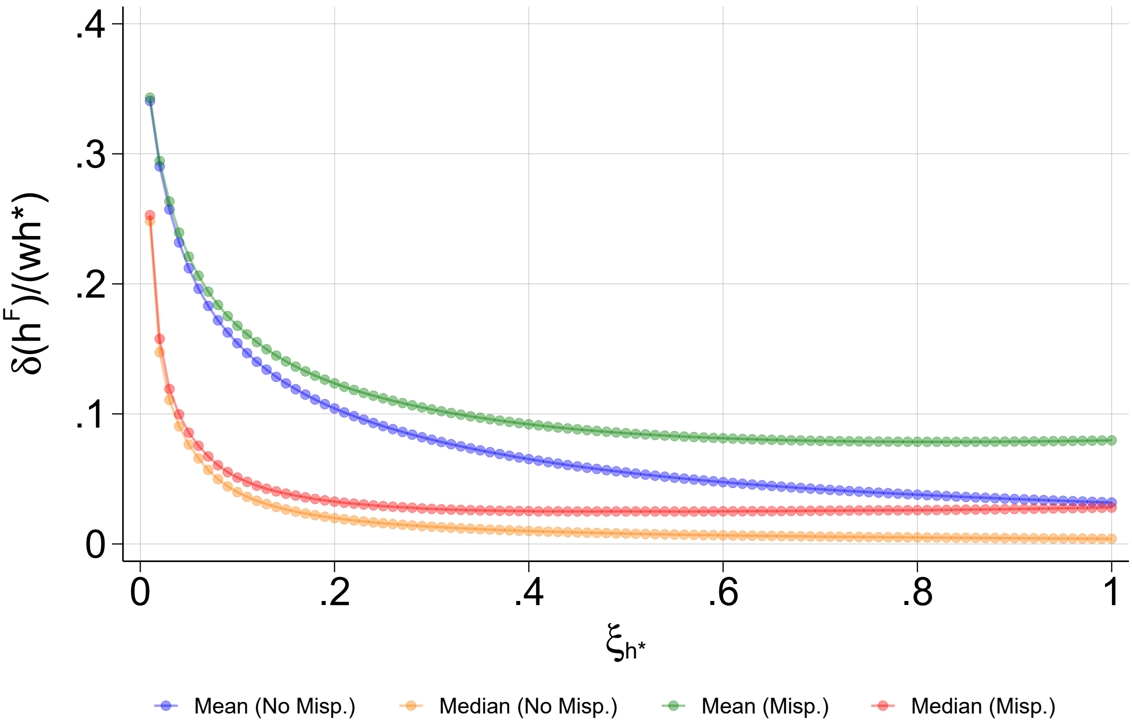
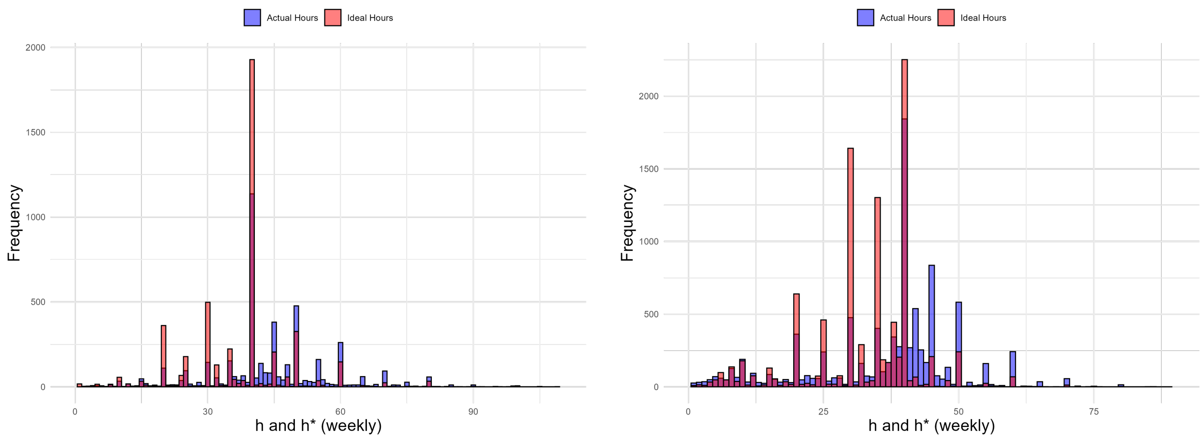


Figure 10: Cumulative Size of Frictions in the GSOEP

Notes: Cost of frictions w/ $\xi_{h^{**}}$.

C Appendix: Chetty (2012)

D Appendix: Figures and Tables

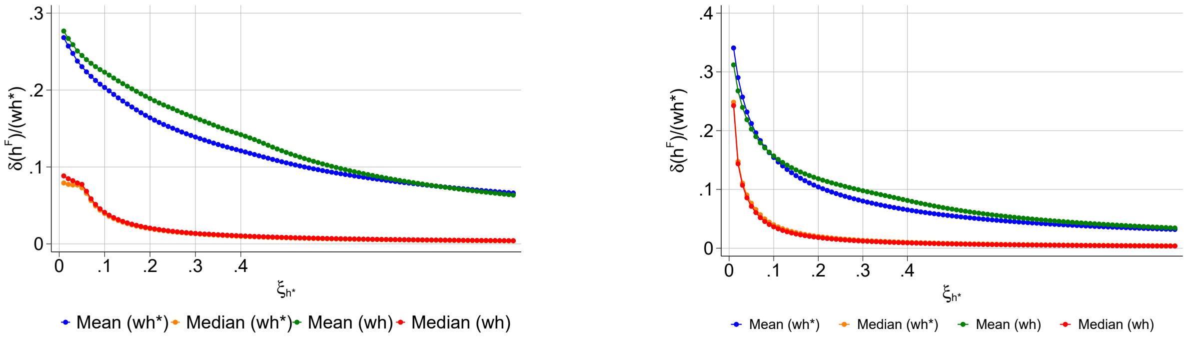


(a) NSCW

(b) GSOEP

Figure 11: h vs. h^* Distributions

Notes: These histograms show the distributions of h and h^* for the NSCW and the GSOEP.



(a) NSCW

(b) GSOEP

Figure 12: $\delta(h^F)/(wh)$ vs. ξ_{h^*}

Notes: This figure shows mean and median values of $\delta(h^F)/(wh)$ as well as $\delta(h^F)/(wh^*)$ for various values of ξ_{h^*} in both the NSCW and the GSOEP.

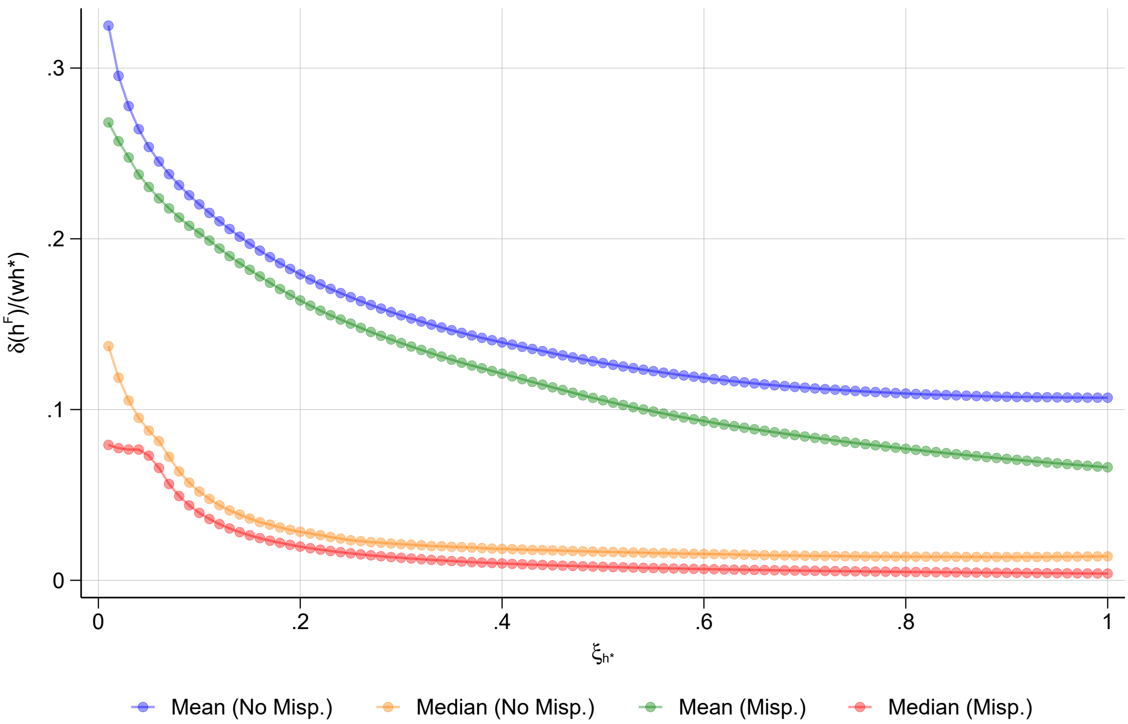
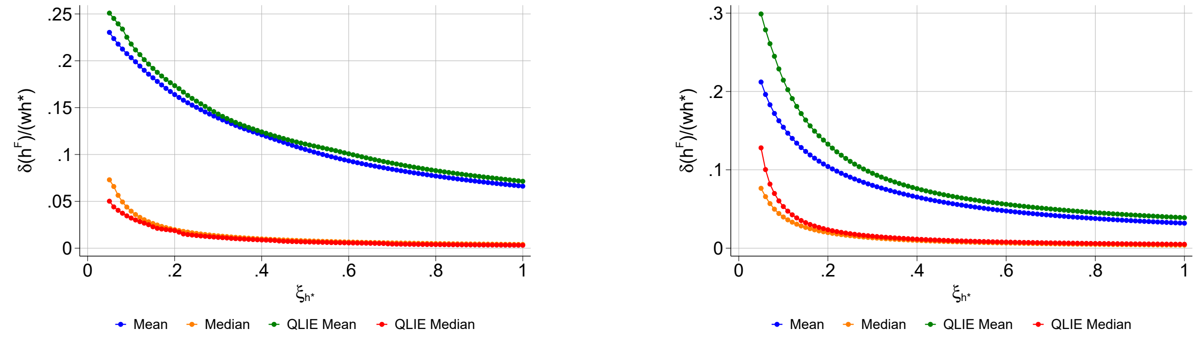


Figure 13: Cost of Adjustment Frictions and Misperceptions, $\eta = -0.5$

Notes: This figure displays the mean and median cumulative cost of frictions along with the mean and median cost of frictions assuming no misperceptions of the tax schedule. We assume $\eta = -0.5$, which means that each exogenous 1% change in after-tax income leads to a 0.5% decrease in ideal hours (see Equation (10)). To simplify computations we also assume that $\log(wh^{**} - \hat{T}(wh^{**})) - \log(wh^{**} - T(wh^{**})) = \log(wh - \hat{T}(wh)) - \log(wh - T(wh))$ so that individuals' misperceptions of their ideal after-tax income are the same as their misperceptions of their actual after-tax income.



(a) NSCW

(b) GSOEP

Figure 14: $\frac{\delta(h^F)}{wh^*}$ vs. ξ_{h^*}

Notes: This figure displays a comparison between the cost of frictions calculated using second order approximation as in Proposition 1 and the cost of frictions calculated assuming that utility is quasi-linear iso-elastic (QLIE) with elasticity ϵ: $u(c,h) = \alpha c - \frac{h^{1+1/\epsilon}}{1+1/\epsilon}$. For each value of ϵ, we calculate the WTP to remove frictions from Equation (2) where α is pinned down from each individuals FOC evaluated at h^*. Note, we still apply Proposition 2 to bound the cost of frictions even if we assume QLIE utility because for some individuals the choice to work much more than their ideal hours rather than not work at all may not be rationalizable with a given elasticity.

D.1 Estimates of the Hours Worked and Taxable Income Elasticities w.r.t. the Marginal Tax Rate

Table 2: Estimates of the Observed Hours Worked Elasticity w.r.t. the Marginal Tax Rate

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Baseline	Cubic	Linear	Time Trends	<50th	Singles	All Hhs	2 Yr Diff	4 Yr Diff
ξ_{h^*}	0.08	0.08	0.11	0.08	0.04	0.29	0.07	-0.01	0.15
	(0.09)	(0.08)	(0.09)	(0.09)	(0.10)	(0.15)	(0.08)	(0.09)	(0.12)
η_{h^*}	0.21	0.18	0.50	0.25	-0.06	0.40	0.29	0.09	0.12
	(0.26)	(0.22)	(0.22)	(0.25)	(0.34)	(0.40)	(0.23)	(0.25)	(0.27)
Obs.	4,328	4,344	4,344	4,328	2,392	2,496	6,229	5,935	3,082

Notes: Standard errors are clustered at the household level and are presented in parentheses. Each column presents estimates from regression (11) except the dependent variable is now the change in log hours worked for individual i, $\Delta \log h_{F,it}$. Columns (1)-(7) use 3 year differences while Columns (8) and (9) use 2 and 4 year differences, respectively. All regressions are for years 1991-2001 inclusive, include year dummies, a married dummy, and restrict to individuals for whom marital status did not change over the pair of differences. All regressions are weighted using household survey weights. Column (1), (4), (5), (6), (7) include a spline in lagged log household income (i.e., lagged income decile dummies interacted with lagged income); Column (4) also includes lagged income interacted with a linear time trend. Instead of a spline, Column (2) includes a cubic polynomial in lagged log household income while Column (3) includes a linear polynomial in lagged log household income. All columns except (6) and (7) restrict to single-earner households. Column (6) restricts to single households only. Column (7) includes both single- and dual-earner households and the dependent variable for Column (7) is now wage weighted observed hours $\Delta \log(w_i h_{it}^F + w_j h_{jt}^F)$ where j represents the second earner (if there is one) and the wages are equal to wages in time period $t-3$. Finally, Column (5) restricts to households with lagged household income below the 50th percentile.

Table 3: Estimates of the Taxable Income Elasticity w.r.t. the Marginal Tax Rate

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Baseline	Cubic	Linear	Time Trends	<50th	Singles	All Hhs	2 Yr Diff	4 Yr Diff
ξ_{h^*}	0.34	0.36	0.54	0.27	0.33	0.67	0.19	0.62	0.50
	(0.19)	(0.18)	(0.18)	(0.20)	(0.20)	(0.31)	(0.15)	(0.16)	(0.21)
η_{h^*}	-0.67	-0.38	0.93	-0.77	-0.55	-0.72	-0.48	0.16	0.21
	(0.38)	(0.35)	(0.35)	(0.39)	(0.59)	(0.61)	(0.29)	(0.36)	(0.43)
Obs.	4,610	4,627	4,627	4,610	2,564	2,673	7,669	6,343	3,255

Notes: Standard errors are clustered at the household level and are presented in parentheses. Each column presents estimates from regression (11) except the dependent variable is now the change in log taxable income for individual i, $\Delta \log w_{it} h_{F,it}$. Columns (1)-(7) use 3 year differences while Columns (8) and (9) use 2 and 4 year differences, respectively. All regressions are for years 1991-2001 inclusive, include year dummies, a married dummy, and restrict to individuals for whom marital status did not change over the pair of differences. All regressions are weighted using household survey weights. Column (1), (4), (5), (6), (7) include a spline in lagged log household income (i.e., lagged income decile dummies interacted with lagged income); Column (4) also includes lagged income interacted with a linear time trend. Instead of a spline, Column (2) includes a cubic polynomial in lagged log household income while Column (3) includes a linear polynomial in lagged log household income. All columns except (6) and (7) restrict to single-earner households. Column (6) restricts to single households only. Column (7) includes both single- and dual-earner households and the dependent variable for Column (7) is now wage weighted observed hours $\Delta \log(w_{it} h^F_{it} + w_{jt} h^F_{jt})$ where j represents the second earner if there is one. Finally, Column (5) restricts to households with lagged household income below the 50th percentile.